MATH 12 SOLUTIONS - SEC. 9.4

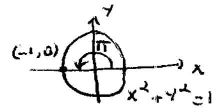
(7) $\sin\theta = -\frac{1}{2}$

1) SOLUTIONS IN $[0, 2\pi)$:

$\theta^* = \frac{\pi}{6}$, so $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$ (Q. III) or $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ (Q. IV) (SINCE $\sin\theta < 0$ IN Q. III AND IV)

2) ALL SOLUTIONS: $\theta = \boxed{\frac{7\pi}{6} + 2n\pi}$ OR $\theta = \boxed{\frac{11\pi}{6} + 2n\pi}$ (WHERE n IS ANY INTEGER)

(9) $\cos\theta = -1$



1) SOLUTIONS IN $[0, 2\pi)$: $\theta = \pi$

2) ALL SOLUTIONS: $\theta = \boxed{\pi + 2n\pi} = \boxed{(2n+1)\pi}$, WHERE n IS ANY INTEGER

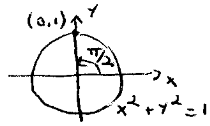
(ALL ODD MULTIPLES OF π)

(14) $2\sin^2 x - 3\sin x + 1 = 0$ SOLUTIONS IN $[0, 2\pi)$:

$(2\sin x - 1)(\sin x - 1) = 0$ $\sin x = \frac{1}{2}$ OR $\sin x = 1$

1) IF $\sin x = \frac{1}{2}$, $x = \boxed{\frac{\pi}{6}}$ (Q. I) OR $x = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$ (Q. II) (SINCE $\sin\theta > 0$ IN Q. I AND II)

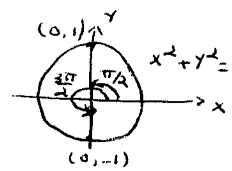
2) IF $\sin x = 1$, $x = \boxed{\frac{\pi}{2}}$



(15) $2\cos^2\theta + \cos\theta = 0$ SOLUTIONS IN $[0, 2\pi)$:

$\cos\theta(2\cos\theta + 1) = 0$ $\cos\theta = 0$ OR $\cos\theta = -\frac{1}{2}$

1) IF $\cos\theta = 0$, $\theta = \boxed{\frac{\pi}{2}}$ OR $\theta = \boxed{\frac{3\pi}{2}}$ USING THE UNIT CIRCLE:



2) IF $\cos\theta = -\frac{1}{2}$, $\theta^* = \frac{\pi}{3}$ so $\theta = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$ (Q. II)

or $\theta = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$ (Q. III) (SINCE $\cos\theta < 0$ IN Q. II AND III)

(19) $2\cos^2 x - \sin x - 1 = 0$ SOLUTIONS IN $[0, 2\pi)$:

$2(1 - \sin^2 x) - \sin x - 1 = 0$

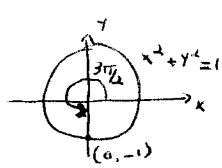
$2 - 2\sin^2 x - \sin x - 1 = 0$

$0 = 2\sin^2 x + \sin x - 1$

$(2\sin x - 1)(\sin x + 1) = 0$ $\sin x = \frac{1}{2}$ OR $\sin x = -1$

1) IF $\sin x = \frac{1}{2}$, $x = \boxed{\frac{\pi}{6}}$ (Q. I) OR $x = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$ (Q. II) (SINCE $\sin\theta > 0$ IN Q. I AND II)

2) IF $\sin x = -1$, $x = \boxed{\frac{3\pi}{2}}$



MATH 12 SOLUTIONS – SEC. 4.5

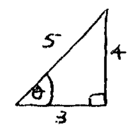
(5) $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\arctan\frac{1}{\sqrt{3}} = \boxed{-\frac{\pi}{6}}$

(11) $\sin\left(\sin^{-1}\frac{1}{4}\right) = \boxed{\frac{1}{4}}$ (since $\sin(\sin^{-1}y) = y$ if $-1 \le y \le 1$)

(14) $\tan(\arctan 3\pi) = \boxed{3\pi}$ (since $\tan(\arctan y) = y$ for any y)

(20) $\sin^{-1}\left(\sin\frac{3\pi}{2}\right) = \sin^{-1}(-1) = -\sin^{-1}1 = \boxed{-\frac{\pi}{2}}$

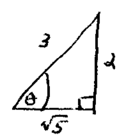
(24) $\tan\left(\sin^{-1}\frac{4}{5}\right)$ Let $\theta = \sin^{-1}\frac{4}{5}$, so $\sin\theta = \frac{4}{5}$:



$= \tan\theta = \boxed{4/3}$

(27) $\sin(\tan^{-1}(-1)) = \sin(-\tan^{-1}1) = \sin\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = \boxed{-\frac{1}{\sqrt{2}}} = \boxed{-\frac{\sqrt{2}}{2}}$

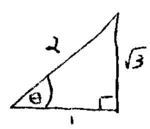
(29) $\cos\left(\sin^{-1}\frac{2}{3}\right)$ Let $\theta = \sin^{-1}\frac{2}{3}$, so $\sin\theta = \frac{2}{3}$:



$= \cos\theta = \boxed{\frac{\sqrt{5}}{3}}$

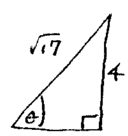
(30) $\cos(\arctan\sqrt{3}) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

OR Let $\theta = \arctan\sqrt{3}$, so $\tan\theta = \sqrt{3}$:



Then $\cos(\arctan\sqrt{3}) = \cos\theta = \boxed{\frac{1}{2}}$

(57) $\sin(2\tan^{-1}4)$ Let $\theta = \tan^{-1}4$, so $\tan\theta = 4$:

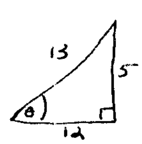


$= \sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{\sqrt{17}}\right)\left(\frac{1}{\sqrt{17}}\right) = \boxed{\frac{8}{17}}$

(58) $\cos\left(2\sin^{-1}\frac{5}{13}\right)$ Let $\theta = \sin^{-1}\frac{5}{13}$, so $\sin\theta = \frac{5}{13}$

$= \cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{5}{13}\right)^2 = 1 - 2\left(\frac{25}{169}\right) = 1 - \frac{50}{169} = \boxed{\frac{119}{169}}$

OR Let $\theta = \sin^{-1}\frac{5}{13}$, so $\sin\theta = \frac{5}{13}$:



Then $\cos\left(2\sin^{-1}\frac{5}{13}\right) = \cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

MATH 12 SOLUTIONS – YELLOW SHEET

(10) $x \cdot \frac{1}{3}(x^2+5)^{-2/3} \cdot 2x + (x^2+5)^{1/3}$

$= \frac{1}{3}(x^2+5)^{-2/3}\left[2x^2 + 3(x^2+5)^1\right] = \frac{1}{3}(x^2+5)^{-2/3}\left[2x^2+3x^2+15\right]$

$= \dfrac{5x^2+15}{3(x^2+5)^{2/3}} = \boxed{\dfrac{5(x^2+3)}{3(x^2+5)^{2/3}}}$

OR $x \cdot \frac{1}{3}(x^2+5)^{-2/3} \cdot 2x + (x^2+5)^{1/3}$

$= \dfrac{2x^2}{3(x^2+5)^{2/3}} + (x^2+5)^{1/3} \cdot \dfrac{3(x^2+5)^{2/3}}{3(x^2+5)^{2/3}}$

$= \dfrac{2x^2}{3(x^2+5)^{2/3}} + \dfrac{3(x^2+5)^1}{3(x^2+5)^{2/3}} = \dfrac{2x^2+3x^2+15}{3(x^2+5)^{2/3}} = \dfrac{5x^2+15}{3(x^2+5)^{2/3}} = \boxed{\dfrac{5(x^2+3)}{3(x^2+5)^{2/3}}}$

(11) $\dfrac{(x^2+1)^2(-2x) - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$

$= \dfrac{2x(x^2+1)\left[-(x^2+1) - 2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}} = \dfrac{2x\left[-x^2-1-2+2x^2\right]}{(x^2+1)^3}$

$= \dfrac{2x(x^2-3)}{(x^2+1)^3} = \boxed{\dfrac{2x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}}$ (since $x^2-3 = x^2-(\sqrt{3})^2 = (x-\sqrt{3})(x+\sqrt{3})$)

(16) $\dfrac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \cdot \dfrac{\sqrt{x+h}\,\sqrt{x}}{\sqrt{x+h}\,\sqrt{x}} = \dfrac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\,\sqrt{x}} \cdot \dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$

$= \dfrac{x-(x+h)}{h\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{\cancel{x}-\cancel{x}-h}{h\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})} = \dfrac{-\cancel{h}}{\cancel{h}\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}$

$= \boxed{\dfrac{-1}{\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}}$

$= \boxed{\dfrac{-1}{x\sqrt{x+h} + (x+h)\sqrt{x}}}$