MATH 12 SOLUTIONS – SEC. 7.4 AND 5.5

7.4 – 8) $\cos\theta = \frac{\sqrt{3}}{2}$ $\quad \theta = \frac{\pi}{6}$ (Q. I) AND $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ (Q. IV)

SOLUTIONS: $\boxed{\theta = \frac{\pi}{6} + 2n\pi,\ \ \theta = \frac{11\pi}{6} + 2n\pi}$ (WHERE n IS ANY INTEGER)

42) $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$: $\bar{\theta} = \frac{\pi}{6}$, SO $\theta = \pi + \frac{\pi}{6} = \boxed{\frac{7\pi}{6}}$ (Q. III) OR $\theta = 2\pi - \frac{\pi}{6} = \boxed{\frac{11\pi}{6}}$ (Q. IV)

OR $\sin\theta = 1$: $\theta = \boxed{\frac{\pi}{2}}$

45) $2\cos^2\theta - 7\cos\theta + 3 = 0$

$(2\cos\theta - 1)(\cos\theta - 3) = 0$

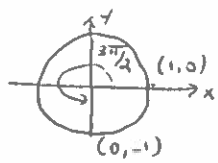
$\cos\theta = \frac{1}{2}$: $\theta = \boxed{\frac{\pi}{3}}$ (Q. I) AND $\theta = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$ (Q. IV)

OR $\cos\theta = 3$: NO SOL., SINCE $-1 \le \cos\theta \le 1$ FOR ALL θ

46) $\sin^2\theta - \sin\theta - 2 = 0$

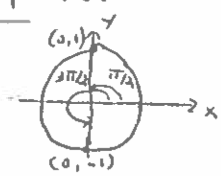
$(\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = -1$: $\theta = \boxed{\frac{3\pi}{2}}$



OR $\sin\theta = 2$: NO SOL., SINCE $-1 \le \sin\theta \le 1$ FOR ALL θ

51) $\cos\theta\,(2\sin\theta + 1) = 0$



$\cos\theta = 0$: $\theta = \boxed{\frac{\pi}{2}}$ OR $\theta = \boxed{\frac{3\pi}{2}}$

OR $\sin\theta = -\frac{1}{2}$: $\theta = \boxed{\frac{7\pi}{6}}$ OR $\theta = \boxed{\frac{11\pi}{6}}$ (AS IN #42)

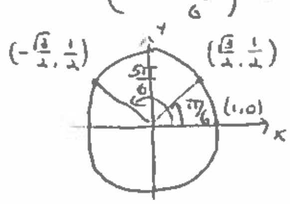
5.5 – 4) a) $\sin^{-1}(-1) = -\sin^{-1} 1 = \boxed{-\frac{\pi}{2}}$ $\qquad$ b) $\sin^{-1}\frac{\sqrt{2}}{2} = \boxed{\frac{\pi}{4}}$

c) $\sin^{-1}(-2)$ IS UNDEFINED, SINCE -2 IS NOT IN $[-1,1]$.

23) $\sin\left(\sin^{-1}\frac{1}{4}\right) = \boxed{\frac{1}{4}}$

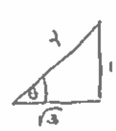
25) $\tan\left(\tan^{-1} 5\right) = \boxed{5}$

33) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \sin^{-1}\frac{1}{2} = \boxed{\frac{\pi}{6}}$ SINCE $\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$ BECAUSE $\bar{\theta} = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$ AND $\sin\theta > 0$ IN Q. II.



(OR USE THIS PICTURE)

39) $\tan\left(\sin^{-1}\frac{1}{2}\right) = \tan\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}}{3}}$



OR LET $\theta = \sin^{-1}\frac{1}{2}$, SO $\sin\theta = \frac{1}{2}$

$\tan\theta = \boxed{\frac{1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}}{3}}$

MATH 12 SOLUTIONS – YELLOW SHEET, SEC. 7.4, SEC. 5.5

Y.S. – 9

$$\frac{x\cdot\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot\cancel{2}x-\sqrt{x^2+1}}{x^2} = \frac{\frac{x^2}{\sqrt{x^2+1}}-\sqrt{x^2+1}}{x^2}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$= \frac{x^2-(x^2+1)}{x^2\sqrt{x^2+1}} = \boxed{\frac{-1}{x^2\sqrt{x^2+1}}}$$

OR

$$\frac{x\cdot\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot\cancel{2}x-\sqrt{x^2+1}}{x^2} = \frac{x^2(x^2+1)^{-1/2}-(x^2+1)^{1/2}}{x^2}$$

$$= \frac{(x^2+1)^{-1/2}\left[x^2-(x^2+1)^1\right]}{x^2} = \boxed{\frac{-1}{x^2(x^2+1)^{1/2}}}$$

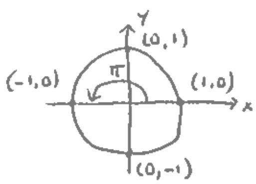
7.4 – 5 $\sin\theta = \frac{\sqrt{3}}{2}$ SOL. IN $[0, 2\pi)$: $\theta = \frac{\pi}{3}$ (Q. I) AND $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (Q. II)

ALL SOLUTIONS : $\boxed{\theta = \frac{\pi}{3} + 2n\pi, \quad \theta = \frac{2\pi}{3} + 2n\pi}$ FOR ANY INTEGER n

7 $\cos\theta = -1$ SOL. IN $[0, 2\pi)$: $\theta = \pi$

ALL SOLUTIONS : $\boxed{\theta = \pi + 2n\pi = (2n+1)\pi}$ FOR ANY INTEGER n

(SO θ IS ANY ODD MULTIPLE OF π)



5.5 – 3 a) $\sin^{-1} 1 = \frac{\pi}{2}$

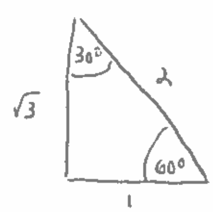
b) $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

c) $\sin^{-1} 2$ IS UNDEFINED (SINCE 2 IS NOT IN $[-1, 1]$)

7 a) $\tan^{-1}(-1) = -\tan^{-1} 1 = -\frac{\pi}{4}$

b) $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

c) $\tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}$



MATH 12 SOLUTIONS - YELLOW SHEET

(10) $x \cdot \frac{1}{3}(x^2+5)^{-2/3} \cdot 2x + (x^2+5)^{1/3}$

$= \frac{1}{3}(x^2+5)^{-2/3}\left[2x^2 + 3(x^2+5)^1\right] = \frac{1}{3}(x^2+5)^{-2/3}\left[2x^2+3x^2+15\right]$

$= \dfrac{5x^2+15}{3(x^2+5)^{2/3}} = \boxed{\dfrac{5(x^2+3)}{3(x^2+5)^{2/3}}}$

OR $x \cdot \frac{1}{3}(x^2+5)^{-2/3} \cdot 2x + (x^2+5)^{1/3}$

$= \dfrac{2x^2}{3(x^2+5)^{2/3}} + (x^2+5)^{1/3} \cdot \dfrac{3(x^2+5)^{2/3}}{3(x^2+5)^{2/3}}$

$= \dfrac{2x^2}{3(x^2+5)^{2/3}} + \dfrac{3(x^2+5)^1}{3(x^2+5)^{2/3}} = \dfrac{2x^2+3x^2+15}{3(x^2+5)^{2/3}} = \dfrac{5x^2+15}{3(x^2+5)^{2/3}} = \boxed{\dfrac{5(x^2+3)}{3(x^2+5)^{2/3}}}$

(11) $\dfrac{(x^2+1)^2(-2x) - (1-x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$

$= \dfrac{2x(x^2+1)\left[-(x^2+1) - 2(1-x^2)\right]}{(x^2+1)^{\cancel{4}\,3}} = \dfrac{2x\left[-x^2-1-2+2x^2\right]}{(x^2+1)^3}$

$= \dfrac{2x(x^2-3)}{(x^2+1)^3} = \boxed{\dfrac{2x(x-\sqrt{3})(x+\sqrt{3})}{(x^2+1)^3}}$ (since $x^2-3 = x^2-(\sqrt{3})^2 = (x-\sqrt{3})(x+\sqrt{3})$)

(16) $\dfrac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \cdot \dfrac{\sqrt{x+h}\,\sqrt{x}}{\sqrt{x+h}\,\sqrt{x}} = \dfrac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\,\sqrt{x}} \cdot \dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$

$= \dfrac{x-(x+h)}{h\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}$

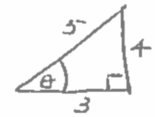
$= \dfrac{\cancel{x} - \cancel{x} - h}{h\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})} = \dfrac{-\cancel{h}}{\cancel{h}\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}$

$= \boxed{\dfrac{-1}{\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x}+\sqrt{x+h})}}$

$= \boxed{\dfrac{-1}{x\sqrt{x+h} + (x+h)\sqrt{x}}}$

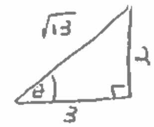
MATH 12 — SOLUTIONS – RED SHEET

1) $\tan\left(\sin^{-1}\frac{4}{5}\right)$ LET $\theta = \sin^{-1}\frac{4}{5}$, SO $\sin\theta = \frac{4}{5}$



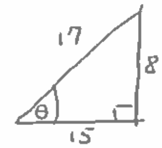
$\tan\theta = \boxed{\frac{4}{3}}$

2) $\sin\left(\tan^{-1}\frac{2}{3}\right)$ LET $\theta = \tan^{-1}\frac{2}{3}$, SO $\tan\theta = \frac{2}{3}$



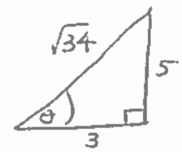
$\sin\theta = \boxed{\frac{2}{\sqrt{13}}}$

3) $\cos\left(\sin^{-1}\frac{8}{17}\right)$ LET $\theta = \sin^{-1}\frac{8}{17}$, SO $\sin\theta = \frac{8}{17}$



$\cos\theta = \boxed{\frac{15}{17}}$

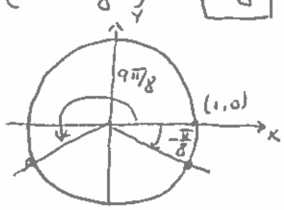
4) $\cos\left(\tan^{-1}\frac{5}{3}\right)$ LET $\theta = \tan^{-1}\frac{5}{3}$, SO $\tan\theta = \frac{5}{3}$



$\cos\theta = \boxed{\frac{3}{\sqrt{34}}}$

5) $\sin^{-1}\left(\sin\frac{9\pi}{8}\right) = \boxed{-\frac{\pi}{8}}$ SINCE $\sin\frac{9\pi}{8} = -\sin\frac{\pi}{8} = \sin\left(-\frac{\pi}{8}\right)$ AND $-\frac{\pi}{8}$ IS IN $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

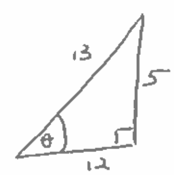
$\left[\theta = \frac{9\pi}{8} - \pi = \frac{\pi}{8},\ \text{AND } \sin\theta < 0 \text{ IN Q. III}\right]$



6) $\sin\left(2\tan^{-1}4\right)$ LET $\theta = \tan^{-1}4$, SO $\tan\theta = 4$

$= \sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{\sqrt{17}}\right)\left(\frac{1}{\sqrt{17}}\right) = \boxed{\frac{8}{17}}$

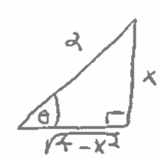
7) $\cos\left(2\sin^{-1}\frac{5}{13}\right)$ LET $\theta = \sin^{-1}\frac{5}{13}$, SO $\sin\theta = \frac{5}{13}$



$= \cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144-25}{169} = \boxed{\frac{119}{169}}$

OR use $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{5}{13}\right)^2 = 1 - 2\cdot\frac{25}{169} = 1 - \frac{50}{169} = \boxed{\frac{119}{169}}$

8) $\sin\theta = \frac{x}{2}$, $0 < \theta < \frac{\pi}{2}$

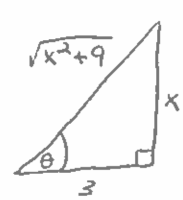


$5\theta - 6\sin 2\theta = 5\theta - 6(2\sin\theta\cos\theta) = 5\sin^{-1}\frac{x}{2} - 12\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right)$

$= \boxed{5\sin^{-1}\frac{x}{2} - 3x\sqrt{4-x^2}}$

SINCE $\theta = \sin^{-1}\frac{x}{2}$ AND $\cos\theta = \frac{\sqrt{4-x^2}}{2}$

9) $\tan\theta = \frac{x}{3}$, $0 < \theta < \frac{\pi}{2}$



$2\theta + 18\sec\theta\tan\theta = 2\tan^{-1}\frac{x}{3} + 18\left(\frac{\sqrt{x^2+9}}{3}\right)\left(\frac{x}{3}\right)$

$= \boxed{2\tan^{-1}\frac{x}{3} + 2x\sqrt{x^2+9}}$

SINCE $\theta = \tan^{-1}\frac{x}{3}$ AND $\sec\theta = \frac{\sqrt{x^2+9}}{3}$