① $\sin\left(x+\frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} = \sin x \cdot 0 + \cos x \cdot 1 = \boxed{\cos x}$

② $\cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$
$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

③ A) $\tan^{-1} 1 = \boxed{\frac{\pi}{4}}$ B) $\tan^{-1}\sqrt{3} = \boxed{\frac{\pi}{3}}$ C) $\tan\left(\arctan\frac{5}{9}\right) = \boxed{\frac{5}{9}}$

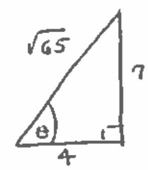
D) $\sin^{-1}\frac{1}{2} = \boxed{\frac{\pi}{6}}$ E) $\sin^{-1}\frac{\sqrt{2}}{2} = \boxed{\frac{\pi}{4}}$ F) $\sin^{-1}\left(\sin\frac{3\pi}{2}\right) = \sin^{-1}(-1) = -\sin^{-1} 1 = \boxed{-\frac{\pi}{2}}$

④ A) $\sin 300^\circ = -\sin 60^\circ = \boxed{-\frac{\sqrt{3}}{2}}$ since $\bar{\theta} = 360^\circ - 300^\circ = 60^\circ$ and $\sin\theta < 0$ in Q. IV.

B) $\cos 240^\circ = -\cos 60^\circ = \boxed{-\frac{1}{2}}$ since $\bar{\theta} = 240^\circ - 180^\circ = 60^\circ$ and $\cos\theta < 0$ in Q. III.

C) $\sin 750^\circ = \sin 30^\circ = \boxed{\frac{1}{2}}$ since $750^\circ - 2(360^\circ) = 30^\circ$

⑤ a) $\underline{\sin 2\theta} = 2\sin\theta\cos\theta = 2\left(\frac{7}{\sqrt{65}}\right)\left(\frac{4}{\sqrt{65}}\right) = \boxed{\frac{56}{65}}$



b) $\underline{\cos 2\theta} = \cos^2\theta - \sin^2\theta = \left(\frac{4}{\sqrt{65}}\right)^2 - \left(\frac{7}{\sqrt{65}}\right)^2 = \frac{16}{65} - \frac{49}{65} = \boxed{-\frac{33}{65}}$

⑥ A) $\boxed{\theta = \frac{3\pi}{2} + 2n\pi}$ for n any integer (since $\theta = \frac{3\pi}{2}$ is the only sol. in $[0, 2\pi)$)

B) $\cos\theta = -\frac{\sqrt{3}}{2}$ $\bar{\theta} = \frac{\pi}{6}$ so $\theta = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$ and $\theta = \pi + \frac{\pi}{6} = \boxed{\frac{7\pi}{6}}$
(since $\cos\theta < 0$ in Q. II and III)

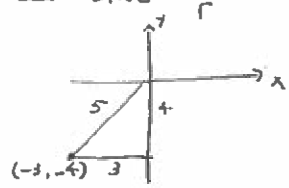
⑦ A) $\sin\theta = -\frac{4}{5}$, $\pi < \theta < \frac{3\pi}{2}$

$\underline{\cos^2\theta} = 1 - \sin^2\theta = 1 - \frac{16}{25} = \underline{\frac{9}{25}}$

so $\cos\theta = \boxed{-\frac{3}{5}}$

(since $\cos\theta < 0$ in Q. III)

OR $\sin\theta = \frac{y}{r} = -\frac{4}{5}$, so let $y = -4$, $r = 5$:



$\underline{\cos\theta = \frac{x}{r}} = \boxed{-\frac{3}{5}}$

B) $\tan\theta = -\frac{5}{12}$, $\frac{3\pi}{2} < \theta < 2\pi$

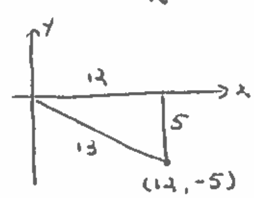
$\underline{\sec^2\theta} = 1 + \tan^2\theta = 1 + \frac{25}{144} = \frac{169}{144}$

so $\underline{\sec\theta} = \frac{13}{12}$ (since $\sec\theta > 0$ in Q. IV)

then $\underline{\cos\theta} = \frac{1}{\sec\theta} = \frac{12}{13}$, so

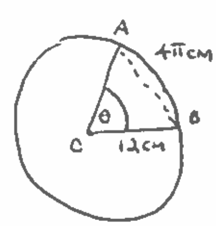
$\underline{\sin\theta} = (\tan\theta)(\cos\theta) = \left(-\frac{5}{12}\right)\left(\frac{12}{13}\right) = \boxed{-\frac{5}{13}}$

OR $\tan\theta = \frac{y}{x} = -\frac{5}{12}$, so let $x = 12$, $y = -5$:



$\underline{\sin\theta = \frac{y}{r}} = \boxed{-\frac{5}{13}}$

⑧ a) $A = \frac{1}{2}ab\sin\theta = \frac{1}{2}(12)(12)\sin\frac{\pi}{3}$
$= \frac{1}{2}\cdot 12\cdot 12\cdot\frac{\sqrt{3}}{2} = \boxed{36\sqrt{3}\ \text{cm}^2}$



$\underline{\theta} = \frac{s}{r} = \frac{4\pi}{12} = \underline{\frac{\pi}{3}}$

b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2\cdot\frac{\pi}{3} = \boxed{24\pi\ \text{cm}^2}$

⑨ $\cos\theta = \frac{1}{8}$, $\frac{3\pi}{2} < \theta < 2\pi$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1+1/8}{2}} = \pm\sqrt{\frac{9/8}{2}} = \pm\sqrt{\frac{9}{16}} = \boxed{-\frac{3}{4}}$

(since $\frac{3\pi}{4} < \frac{\theta}{2} < \pi$, and $\cos x < 0$ in Q. II)

(10) $\sin^2 3\theta \cos^2 3\theta = \frac{1}{2}(1-\cos 6\theta)\cdot\frac{1}{2}(1+\cos 6\theta) = \frac{1}{4}(1-\cos^2 6\theta) = \frac{1}{4}\sin^2 6\theta = \frac{1}{4}\cdot\frac{1}{2}(1-\cos 12\theta)$

$= \boxed{\frac{1}{8}(1-\cos 12\theta)}$

(11) a) $x^4\cdot\frac{1}{2}(5x+9)^{-1/2}\cdot 5 + 4x^3\sqrt{5x+9}$

$= \frac{1}{2}x^3(5x+9)^{-1/2}\left[5x + 8(5x+9)\right]$

$= \frac{1}{2}x^3(5x+9)^{-1/2}\left[45x+72\right]$

$= \frac{1}{2}x^3(5x+9)^{-1/2}\cdot 9(5x+8)$

$= \boxed{\dfrac{9x^3(5x+8)}{2(5x+9)^{1/2}}}$

OR $x^4\cdot\frac{1}{2}(5x+9)^{-1/2}\cdot 5 + 4x^3\sqrt{5x+9}$

$= \dfrac{5x^4}{2\sqrt{5x+9}} + 4x^3\sqrt{5x+9}\cdot\dfrac{2\sqrt{5x+9}}{2\sqrt{5x+9}}$

$= \dfrac{5x^4+8x^3(5x+9)}{2\sqrt{5x+9}} = \dfrac{5x^4+40x^4+72x^3}{2\sqrt{5x+9}}$

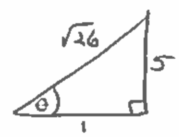
$= \dfrac{45x^4+72x^3}{2\sqrt{5x+9}} = \boxed{\dfrac{9x^3(5x+8)}{2\sqrt{5x+9}}}$

B) $\dfrac{(x^2+75)^2\cdot 4 - 4x\cdot 2(x^2+75)\cdot 2x}{(x^2+75)^4} = \dfrac{4(x^2+75)\left[x^2+75-4x^2\right]}{(x^2+75)^{\cancel{4}\,3}}$

$= \dfrac{4\left[75-3x^2\right]}{(x^2+75)^3} = \dfrac{4\cdot 3(25-x^2)}{(x^2+75)^3} = \boxed{\dfrac{12(5-x)(5+x)}{(x^2+75)^3}} = \boxed{\dfrac{-12(x-5)(x+5)}{(x^2+75)^3}}$

(12) $\sin(2\tan^{-1}5)$ LET $\theta = \tan^{-1}5$, SO $\tan\theta = 5$

$= \sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{5}{\sqrt{26}}\right)\left(\frac{1}{\sqrt{26}}\right) = \frac{10}{26} = \boxed{\frac{5}{13}}$



(13) IF $x = 5\sin\theta$ AND $0<\theta<\frac{\pi}{2}$,

$\dfrac{x}{\sqrt{25-x^2}} = \dfrac{5\sin\theta}{\sqrt{25-25\sin^2\theta}} = \dfrac{5\sin\theta}{\sqrt{25(1-\sin^2\theta)}} = \dfrac{5\sin\theta}{\sqrt{25\cos^2\theta}} = \dfrac{5\sin\theta}{5|\cos\theta|} = \dfrac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$

(SINCE $\cos\theta > 0$ IN Q. I)

(14) a) $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$, AND $0\le x\le 2\pi \Rightarrow 0\le\frac{x}{2}\le\pi \Rightarrow \sin\frac{x}{2}\ge 0$,

SO $\sin\frac{x}{2} = +\sqrt{\frac{1-\cos x}{2}} = \frac{\sqrt{1-\cos x}}{\sqrt{2}}$ AND $\sqrt{1-\cos x} = \boxed{\sqrt{2}\sin\frac{x}{2}}$

b) LET θ_1 BE THE ANGLE OF INCLINATION OF $y=2x+1$, SO $\tan\theta_1 = 2$

LET θ_2 BE THE ANGLE OF INCLINATION OF $y=5x-17$, SO $\tan\theta_2 = 5$

THEN $\tan\theta = \tan(\theta_2-\theta_1) = \dfrac{\tan\theta_2-\tan\theta_1}{1+\tan\theta_2\tan\theta_1} = \dfrac{5-2}{1+5(2)} = \dfrac{3}{11}$

SO $\theta = \boxed{\tan^{-1}\frac{3}{11}}$