Math 135A | Midterm | Fall 2007

NAME(print in CAPITAL letters, first name first): KEY

NAME(sign): ______________________________

ID#: ______________________________

Instructions: There are five problems. Some questions are easier than others so you are encouraged to read the entire exam before beginning your work. Make sure that you have a total of 6 pages (including this one) with 5 problems.

1	
2	
3	
4	
5	
TOTAL	

1. A fair six-sided die is rolled twice. Find the distribution of the maximum of the two rolls.

roll 2 \ roll 1	1	2	3	4	5	6
6	6	6	6	6	6	6
5	5	5	5	5	5	6
4	4	4	4	4	5	6
3	3	3	3	4	5	6
2	2	2	3	4	5	6
1	1	2	3	4	5	6

← max

$$P(\max = 1) = \frac{1}{36}$$

$$P(\max = 2) = \frac{3}{36}$$

$$P(\max = 3) = \frac{5}{36}$$

$$\vdots$$

$$\boxed{P(\max = k) = \frac{2k-1}{36}, \quad k = 1, 2, \ldots, 6}$$

2. A Biology class consists of 80% women and 20% men. Among these, 10% of the men and 1% of the women are failing the class. A student is chosen uniformly at random. Given that the student is failing, what is the conditional probability that it is a man?

$$P(\text{man} \mid \text{Fail}) = \frac{P(\text{man}, \text{Fail})}{P(\text{Fail})}$$

$$= \frac{P(\text{man})\, P(\text{Fail} \mid \text{man})}{P(\text{man}) P(\text{Fail} \mid \text{man}) + P(\text{man}^c) P(\text{Fail} \mid \text{man}^c)}$$

$\text{man}^c = \text{woman}$

$$= \frac{(0.2)(0.1)}{(0.2)(0.1) + (0.8)(0.01)}$$

$$= \frac{0.02}{0.02 + 0.008}$$

$$= \frac{20}{20+8} = \boxed{\frac{5}{7}}$$

3. A fair coin is flipped ten times.

   (a) Given that six of the ten flips were heads, what is the conditional probability that three of the first five were heads?

$$P(3 \text{ in } 5 \mid 6 \text{ in } 10) = \frac{P(3 \text{ in } 5, 6 \text{ in } 10)}{P(6 \text{ in } 10)}$$

$$= \frac{P(3 \text{ in } 5, 3 \text{ in next } 5)}{P(6 \text{ in } 10)}$$

$$= \frac{\left(\binom{5}{3}\left(\frac{1}{2}\right)^5\right)^2}{\binom{10}{6}\left(\frac{1}{2}\right)^{10}} = \boxed{\frac{\binom{5}{3}^2}{\binom{10}{6}}}$$

   (b) Given that at least nine of the flips were heads, what is the conditional probability that all ten of the flips were heads?

$$P(10 \text{ heads} \mid \text{at least } 9 \text{ heads}) =$$

$$\frac{P(10 \text{ heads, at least } 9 \text{ heads})}{P(\text{at least } 9)}$$

$$= \frac{P(10 \text{ heads})}{P(\text{at least } 9)}$$

$$= \frac{\left(\frac{1}{2}\right)^{10}}{\underbrace{\binom{10}{1}\left(\frac{1}{2}\right)^{10}}_{P(9)} + \underbrace{\left(\frac{1}{2}\right)^{10}}_{P(10)}} = \boxed{\frac{1}{11}}$$

4. A five card poker hand is dealt from a standard deck. Find the probability that the hand is three of a kind (ranks a, a, a, b, c).

choose ~~ranks:~~

choices for triple: 13

choices for singles: $\binom{12}{2}$

(not $12 \cdot 11$ b/c order doesn't matter)

choose suits

choices for triple: $\binom{4}{3}$

choices for lower rank single: $\binom{4}{1}$

choices for higher rank single: $\binom{4}{1}$

so the answer is

$$\frac{13\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$$

5. A fair die is rolled ten times. Find the expected number of faces that appear at least once.

$$X = X_1 + \cdots + X_6$$

where

$$X_j = \begin{cases} 1 & \text{if face } j \text{ appears} \\ 0 & \text{else} \end{cases}$$

$$E(X) = E(X_1) + \cdots + E(X_6) = 6\,E(X_1)$$

$$E(X_1) = P(\text{face } 1 \text{ appears}) = 1 - P(\text{face } 1 \text{ doesn't appear}) = 1 - \left(\frac{5}{6}\right)^{10}$$

So

$$E(X) = 6\left(1 - \left(\frac{5}{6}\right)^{10}\right)$$