Homework 3

due October 23, 2001

1. Prove that $\mathrm{SL}_n(\mathbb{R}) \trianglelefteq \mathrm{GL}_n(\mathbb{R})$. Here $\mathrm{SL}_n(\mathbb{R})$ is the special linear group defined as
$$\mathrm{SL}_n(\mathbb{R}) = \{A \in \mathrm{GL}_n(\mathbb{R}) \mid \det A = 1\}.$$
2. Find the center of $\mathrm{GL}_2(\mathbb{R})$.
3. Let G be a group. Show that if $G/Z(G)$ is cyclic then G is abelian.
4. Let $H \leq G$ with index $[G : H] = n$. Show that G has a normal subgroup of index at most $n!$.
(Hint: Consider the action of G on the left cosets of H).
5. Let G be a group, $N \trianglelefteq G$ and let $\bar{G} = G/N$. Prove that $\bar{x}, \bar{y} \in \bar{G}$ commute if and only if $x^{-1}y^{-1}xy \in N$. The element $x^{-1}y^{-1}xy$ is called the commutator of x and y denoted by $[x, y]$.
6. Show that all subgroups of index 2 are normal.
7. Find all normal subgroups of S_4.
8. Dummit, Foote Section 2.1 Exercise 6 (page 49)
9. Dummit, Foote Section 3.1 Exercise 14 (page 87)