Homework 7

due November 19, 2003

1. Rosen 6.1 #40,41, pg. 204
Using the fact that p divides the binomial coefficient $\binom{p}{k}$ when $0 < k < p$, show that if a, b are integers then $(a+b)^p \equiv a^p + b^p \pmod{p}$. Use this to prove Fermat's little theorem.

2. Rosen 6.2 #2, pg. 213
Show that 45 is a pseudoprime to the bases 17 and 19.

3. Rosen 6.2 #20, pg. 214
Show that if n is a Carmichael number then n is squarefree.

4. Rosen 6.3 #6, pg. 218
Find the last digit of the decimal expansion of $7^{999,999}$.

5. Rosen 6.3 #10, pg. 218
Show that $a^{\Phi(b)} + b^{\Phi(a)} \equiv 1 \pmod{ab}$, if a and b are relatively prime positive integers.

6. Rosen 7.1 #8, pg. 228
Show that there is no positive integer n such that $\Phi(n) = 14$.

7. Rosen 7.1 #18, pg. 228
Show that if m and k are positive integers, then $\Phi(m^k) = m^{k-1}\Phi(m)$.