Homework Set Two: Fundamental Theorem of Algebra and Vector Spaces

Directions: Submit your solutions to the Calculational Exercises and the Proof-Writing Exercises **separately** at the **beginning** of lecture on **Friday, October 12, 2007**. The two problems sets will be graded by different persons.

Calculational Exercises

1. Given any complex number $\alpha \in \mathbb{C}$, show that the coefficients of the polynomial

$$(z-\alpha)(z-\overline{\alpha})$$

are real numbers.

2. Show that the space $V = \{(x_1, x_2, x_3) \in \mathbb{F}^3 \mid x_1 + 2x_2 + 2x_3 = 0\}$ forms a vector space.

3. Give an example of a nonempty subset $U \subset \mathbb{R}^2$ such that U is closed under scalar multiplication but is not a subspace of $\mathbb{R}^2$.

Proof-Writing Exercises

1. Let $p(z)$ be a polynomial with real coefficients, and let $\alpha \in \mathbb{C}$ be a complex number. Prove that $p(\alpha) = 0$ if and only $p(\overline{\alpha}) = 0$.

2. Let V be a vector space over $\mathbb{F}$. Then, given $a \in \mathbb{F}$ and $v \in V$ such that $av = 0$, prove that either $a = 0$ or $v = 0$.

3. Let V be a vector space over $\mathbb{F}$, and suppose that W_1 and W_2 are subspaces of V. Prove that their intersection $W_1 \cap W_2$ is also a subspace of V.