

Syllabus: Advanced Linear Algebra

L #	Topics	Notes & Remarks
1	What is linear algebra?	Chapter 1
2,3	Complex numbers	Chapter 2
	Discussion: Calculations with complex numbers; encoding linear systems	covers L1–3; 12.1
4	Fundamental theorem of algebra (proof optional)	Chapter 3
5	Vector spaces and subspaces	Chapter 4.1–4.3
6	Direct sum, linear span	Chapter 4.4–5.1
	Discussion: Vector space of matrices and operations on matrices	covers L5,6; 12.2
7	Linear independence of vectors	Chapter 5.2
8	Bases and dimensions of vector spaces	Chapter 5.3–5.4
9	Linear maps	Chapter 6.1
	Discussion: Linear independence, homogenous linear systems, Gaussian elimination	covers: L7,8; 12.3.1–2
10	Null space and range of linear maps	Chapter 6.2–6.4
11	Dimension formula for a linear map	Chapter 6.5
12	Matrix of a linear map	Chapter 6.6
	Discussion: Linear maps, inhomogeneous systems, LU-factorization	covers L9–12; 12.3.3–4
13	Invertibility	Chapter 6.7
14	Midterm	
15	Eigenvalues and eigenvectors	Chapter 7.1–7.3
	Discussion: Linear maps	L12,13; 12.4
16	Existence of eigenvalues	Chapter 7.4
17	Upper triangular matrix representation	Chapter 7.5
18	Diagonalization (2x2) and applications	Chapter 7.6
	Discussion: Eigenvalues and eigenvectors, special operations on matrices	L15–18; 12.5
19	Permutations and the determinant	Chapter 8.1–8.5
20	Properties of the determinant	Chapter 8.6–8.7
21	Inner product spaces	Chapter 9.1–9.2
	Discussion: Calculation of the determinant, inner product spaces	L19–21
22	Cauchy-Schwarz, triangle inequality, Pythagoras	Chapter 9.3
23	Orthonormal bases, Gram-Schmidt procedure	Chapter 9.4–9.5
24	Orthogonal projections, minimization problems	Chapter 9.6
	Discussion: Gram-Schmidt procedure and orthogonal projections	L22–24
25	Change of bases	Chapter 10
26	Self-adjoint and normal operators	Chapter 11.1–11.2
27	Spectral theorem for normal maps (complex)	Chapter 11.3
	Discussion: Change of basis, diagonalization	L25–27
28	Diagonalization	Chapter 11.4
29	Positive operators, polar and singular value decompositions	Chapter 11.6–11.7

Lecture notes *Linear Algebra as an Introduction to Abstract Mathematics* by Isaiah Lankham, Bruno Nachtergaele, and Anne Schilling are available on the class website at

<http://www.math.ucdavis.edu/~anne/FQ2007/mat67.html>