Homework Set Five: Matrices and Linear Maps

Directions: Submit your solutions to the homework at the **beginning** of lecture on **Friday, October 30, 2009**.

Calculational Exercises

1. Define the map $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x, y) = (x + y, x)$.
   (a) Show that T is linear.
   (b) Show that T is surjective.
   (c) Find $\dim(\mathrm{null}\,(T))$.
   (d) Find the matrix for T with respect to the canonical basis of $\mathbb{R}^2$.
   (e) Find the matrix for T with respect to the canoncial basis for the domain $\mathbb{R}^2$ and the basis $((1, 1), (1, -1))$ for the target space $\mathbb{R}^2$.
   (f) Show that the map $F : \mathbb{R}^2 \to \mathbb{R}^2$ given by $F(x, y) = (x + y, x + 1)$ is not linear.

2. Consider the complex vector spaces $\mathbb{C}^2$ and $\mathbb{C}^3$ with their canonical bases, and define $S \in \mathcal{L}(\mathbb{C}^3, \mathbb{C}^2)$ be the linear map defined by $S(v) = Av, \forall v \in \mathbb{C}^3$, where A is the matrix

$$A = M(S) = \begin{pmatrix} i & 1 & 1 \\ 2i & -1 & -1 \end{pmatrix}.$$

   Find a basis for $\mathrm{null}(S)$.

Proof-Writing Exercises

1. Let V be a finite-dimensional vector space over $\mathbb{F}$ with $S, T \in \mathcal{L}(V, V)$. Prove that $T \circ S$ is invertible if and only if both S and T are invertible.