Homework 7

due November 17, 2010

1. Rosen 6.3 #6, pg. 236
Find the last digit of the decimal expansion of $7^{999,999}$.

2. Rosen 6.3 #10, pg. 236
Show that $a^{\Phi(b)} + b^{\Phi(a)} \equiv 1 \pmod{ab}$, if a and b are relatively prime positive integers.

3. Rosen 6.3 #12, pg. 236
Show that the solutions to the simultaneous system of congruences

$$\begin{aligned} x &\equiv a_1 \pmod{m_1} \\ x &\equiv a_2 \pmod{m_2} \\ &\cdots \\ x &\equiv a_r \pmod{m_r} \end{aligned}$$

where m_j are pairwise relatively prime, are given by

$$x \equiv a_1 M_1^{\Phi(m_1)} + \cdots + a_r M_r^{\Phi(m_r)} \pmod{M},$$

where $M = m_1 m_2 \cdots m_r$ and $M_j = M/m_j$ for $j = 1, 2, \ldots, r$.

4. Rosen 7.1 #8, pg. 245
Show that there is no positive integer n such that $\Phi(n) = 14$.

5. Rosen 7.1 #22, pg. 246
Show that if m and k are positive integers, then $\Phi(m^k) = m^{k-1}\Phi(m)$.

6. Rosen 7.2 #11, pg. 253
What is the product of the positive divisors of a positive integer n?

7. Rosen 7.2 #12, pg. 253
Show that the equation $\sigma(n) = k$ has at most a finite number of solutions when k is a positive integer.