Homework 8

due December 2, 2011 in class

Read: Artin 5.5-5.7, 6.1-6.6

(1) **Artin 5.5.5** pg. 193
Let $G = D_4$ be the dihedral group of symmetries of the square.
(a) What is the stabilizer of a vertex? an edge?
(b) G acts on the set of two elements consisting of the diagonal lines. What is the stabilizer of a diagonal?

(2) **Artin 5.6.7** pg. 194
A map $\varphi : S \to S'$ of G-sets is called a *homomorphism* of G-sets if $\varphi(gs) = g\varphi(s)$ for all $s \in S$ and $g \in G$. Let φ be such a homomorphism. Prove the following:
(a) The stabilizer $G_{\varphi(s)}$ contains the stabilizer G_s.
(b) The orbit of an element $s \in S$ maps onto the orbit of $\varphi(s)$.

(3) Find the number of distinguishable ways the edges of a square can be painted if six colors of paint are available and the same color may be used on more than one edge.

(4) Decide if the following statements are **true** or **false**. Briefly justify your response.
(a) Every G-set is also a group.
(b) Let S be a G-set with $s_1, s_2 \in S$ and $g \in G$. If $gs_1 = gs_2$, then $s_1 = s_2$.
(c) Let S be a G-set with $s \in S$ and $g_1, g_2 \in G$. If $g_1 s = g_2 s$, then $g_1 = g_2$.

(5) **Artin 6.1.2** pg. 229
Let H be a subgroup of a group G. Then H operates on G by left multiplication. Describe the orbits for this operation.

(6) **Artin 6.1.6** pg. 229
Rule out as many of the following as possible as Class Equations for a group of order 10:
1+1+1+2+5, 1+2+2+5, 1+2+3+4, 1+1+2+2+2+2.

(7) **Artin 6.4.2** pg. 231
Prove that no group of order pq, where p and q are prime, is simple.

(8) Let G be a finite group and let $P \leq G$ be a Sylow p-subgroup of G. Show that P is the unique Sylow p-subgroup if and only if P is normal in G.