**Homework 2**

due October 9, 2015 in class

**Read:** Artin Chapters 2.3, 2.4, 2.5

1. Artin 2.1.1 (pg. 69)

   Let $S$ be a set. Prove that the law of composition defined by $ab = a$ for all $a, b \in S$ is associative. For which sets does this law have an identity?

2. Artin 2.2.6 (pg. 70)

   Let $G$ be a group, with multiplicative notation. We define an *opposite group* $G^0$ with law of composition $a \star b$ as follows: The underlying set is the same as $G$, but the law of composition is the opposite; that is, we define $a \star b = ba$. Prove that this defines a group.

3. Determine the elements of the cyclic group generated by $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ explicitly.

4. Artin 2.4.1 (pg. 70)

   Let $a, b$ be elements of a group $G$. Assume that $a$ has order 7 and that $a^3b = ba^3$. Prove that $ab = ba$.

5. Artin 2.2.4bcde (pg. 70)

   In which of the following cases is $H$ a subgroup of $G$?

   **(b)** $G = \mathbb{R}^\times$ and $H = \{1, -1\}$.

   **(c)** $G = \mathbb{Z}^+$ and $H$ is the set of positive integers.

   **(d)** $G = \mathbb{R}^\times$ and $H$ is the set of positive reals.

   **(e)** $G = GL_2(\mathbb{R})$ and $H$ is the set of all matrices $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$ with $a \neq 0$.

6. Artin 2.4.5 (pg. 70)

   Prove that every subgroup of a cyclic group is cyclic.

7. Artin 2.4.6 (pg. 70)

   **(a)** Let $G$ be a cyclic group of order 6. How many of its elements generate $G$?

   **(b)** Answer the same question for cyclic groups of order 5 and 8.

   **(c)** Describe the number of elements that generate a cyclic group of arbitrary order $n$.

8. Prove that a group in which every element except the identity has order 2 is abelian.