Homework 5
due November 6, 2015 in class

Read: Artin 2.11-2.12

(1) Artin 2.11.6 (pg. 74)
Let G be a group containing normal subgroups of orders 3 and 5, respectively. Prove that G contains an element of order 15.

(2) Let G be a finite group whose order is a product of two integers: $n = ab$. Let H, K be subgroups of G of orders a and b respectively. Assume that $H \cap K = \{1\}$. Prove that $HK = G$. Is G isomorphic to the product group $H \times K$?

(3) (a) Prove 2 has no inverse modulo 6.
(b) Determine all integers n such that 2 has an inverse modulo n.

(4) Prove that the subset H of $G = GL_n(\mathbb{R})$ of matrices whose determinant is positive forms a normal subgroup, and describe the quotient group G/H.

(5) Prove that the subset $G \times 1$ of the product group $G \times G'$ is a normal subgroup isomorphic to G and that $(G \times G')/(G \times 1)$ is isomorphic to G'.

(6) Artin 2.M.2(a) (pg. 75)
Prove that a group of even order contains an element of order 2.

(7) Let $K \subset H \subset G$ be subgroups of a finite group G. Prove $[G : K] = [G : H][H : K]$.