Homework 8

due Wednesday November 25, 2015 in class

Read: Artin 6.5, 6.7

1. Let G be a discrete subgroup of $M := \mathrm{Iso}(\mathbb{R}^2)$. Show that every subgroup of G is discrete.

2. Prove that a discrete group G consisting of rotations about the origin is cyclic and is generated by ρ_θ where θ is the smallest angle of rotation in G.

3. Let G be a subgroup of M which contains rotations about two different points. Prove algebraically that G contains a translation.
   Hint: Write the two rotations as $t_a\rho_\theta$ and $t_b\rho_\eta$ and consider
   $$(t_a\rho_\theta)(t_b\rho_\eta)(t_a\rho_\theta)^{-1}(t_b\rho_\eta)^{-1}.$$

4. Prove that every discrete subgroup of O_2 is finite.

5. A group G acts **transitively** on a non-empty G-set S if, for all $s_1, s_2 \in S$, there exists an element $g \in G$ such that $gs_1 = s_2$. Characterize transitive G-set actions in terms of orbits. Prove your answer.

6. A group G acts **faithfully** on a G-set S if $gs = s$ for all $s \in S$ implies $g = 1$. Show that G acts faithfully on S if and only if no two distinct elements of G have the same action on every element of S.