MAT 146 University of California Spring 2014

Homework 6

due Friday May 23, 2014 in class

1. Stanley, Chapter 5.5
A $(0,1)$-necklace of length n and weight i is a circular arrangement of i 1's and $n-i$ 0's. For instance, the $(0,1)$-necklaces of length 6 and weight 3 are (writing a circular arrangement linearly) 000111, 001011, 010011, and 010101. (Cyclic shifts of a linear word represent the same necklace, e.g., 000111 is the same as 110001.) Let N_n denote the set of all $(0,1)$-necklaces of length n. Define a partial order on N_n by letting $u \leq v$ if we can obtain v from u by changing some of the 0's to 1's. It is easy to see (you may assume it) that N_n is graded of rank n, with the rank of a necklace being its weight.

1. Show that N_n is rank-symmetric, rank-unimodal, and Sperner.
   Hint: Show that the $(0,1)$-necklace poset is isomorphic to B_n/G for a suitable subgroup G of S_n.
2. (not assigned, will be done in class, but if you can do this one yourself you are ready for research!) Show that N_n has a symmetric chain decomposition.

2. Stanley, Chapter 5.7
Let M be a finite multiset, say with a_i i's for $1 \leq i \leq k$. Let B_M denote the poset of all submultisets of M, ordered by multiset inclusion. For instance, for $a_1 = 2$, $a_2 = 3$ we have $M = 11222$. Then the elements in B_M are $\emptyset, 1, 2, 11, 12, 22, 112, 122, 222, 1122, 1222, 11222$ and for example $122 \leq 1222$.

Use Theorem 5.8 to show that B_M is rank-symmetric, rank-unimodal, and Sperner. (There are other ways to do this problem, but you are asked to use Theorem 5.8. Thus you need to find a subgroup G of S_n for suitable n for which $B_M \cong B_n/G$).