Homework 2

due January 23, 2002 in class

1. Artin 6.1.2 (pg. 229)
2. Artin 6.1.6 (pg. 229)
3. Artin 6.4.2 (pg. 231)
4. Let G be a group with center Z and for each $g \in G$, let $Z(g)$ denote the centralizer of g. Show that $g \in Z$ if and only if $Z(g) = G$.
5. Let G be a finite group and let $P \leq G$ be a Sylow p-subgroup of G. Show that P is the unique Sylow p-subgroup if and only if P is normal in G.
6. Let G be a group, $H \leq G$ and $N(H)$ be the normalizer of H in G. Show that $N(H)$ is the largest subgroup of G containing H as a normal subgroup.
7. Let G be a finite group. Show that the number of 1's in the class equation of G is exactly the order of the center of G.
8. If H is a normal subgroup of order p^k of a finite group G, then H is contained in every Sylow p-subgroup of G.