Homework 8
due March 6, 2002 in class

Read Chapter 7.3 in Artin.

(1) Let $G = \mathrm{GL}_n(\mathbb{R})$ and let $S = M_n(\mathbb{R})$ be the set of all $n \times n$ matrices over $\mathbb{R}$. Show that the map $G \times S \to S$ given by

$$(P, A) \mapsto (P^t)^{-1} A P^{-1}$$

defines an action of G on S.

(2) Let $G = \mathrm{GL}_n(\mathbb{C})$ and let $S = M_n(\mathbb{C})$ be the set of all $n \times n$ matrices over $\mathbb{C}$. Show that the map $G \times S \to S$ given by

$$(P, A) \mapsto (P^*)^{-1} A P^{-1}$$

defines an action of G on S.

(3) Find the stabilizer of the identity matrix I_n under the action of $\mathrm{GL}_n(\mathbb{R})$ on $M_n(\mathbb{R})$ given in Problem 1.

(4) Find the stabilizer of the identity matrix I_n under the action of $\mathrm{GL}_n(\mathbb{C})$ on $M_n(\mathbb{C})$ given in Problem 2.

(5) Show that the product AA^* is hermitian for all $n \times m$ complex matrices A.

(6) Artin 7.4.4 (pg. 265)

(7) Artin 7.5.3 (pg. 266)

(8) Artin 7.5.15 (pg. 267)