

Sample Final Problems

The format of the final will be similar to the midterm. There will be 8 questions. The first two questions will be definitions and true and false statements. The remaining questions will be similar to the homework problems and contain statements that you are asked to prove.

Material covered:

The exam is comprehensive covering all material that was discussed in class. Make sure that you understand the solutions of the homework problems (the solutions are on the class homepage). Here is a list of things you should know and types of questions that may be asked:

- Definitions: learn them all!
- Show a map $G \times S \rightarrow S$ is a group action.
- Find orders of stabilizers, orbits, etc..
- Applications of Burnside's theorem, Sylow theorems, conjugation.
- Relationship between isometries of \mathbb{R}^n and orthogonal operators.
- Showing properties of finite and discrete subgroups of $\text{Iso}(\mathbb{R}^2)$.
- Determine the symmetry group of a figure in \mathbb{R}^2 . What is the translation group L_G , or its point group \overline{G} .
- Properties of positive definite, symmetric bilinear forms and hermitian forms. Applications of the Spectral Theorem.
- Find the matrix $\varphi(P)$ for a given $P \in SU_2$.
- Find the matrix representation of a representation $\rho : G \rightarrow \text{GL}(V)$ in a given basis.

Sample questions:

- (1) Suppose a finite group G acts on a finite set S . Show that if $x, y \in S$ lie in the same orbit, then $|G_x| = |G_y|$, where G_x denotes the stabilizer of $x \in S$.
- (2) How many striped flags are there having six stripes (of equal width) each of which can be colored red, white or blue?
- (3) Let Q be a normal p -subgroup of a finite group G . Show that $Q \leq P$ for every Sylow p -subgroup P of G .
- (4) Show that if G is a non-trivial discrete subgroup of $\text{Iso}(\mathbb{R}^2)$ consisting of rotations about the origin, then G is isomorphic to a finite cyclic group \mathbb{Z}_n for some n .
- (5) Show that an element $A \in M_n(\mathbb{C})$ is skew-hermitian if and only if $H = iA$ is hermitian.