

Problem: Find the number of different ways to color a cube such that 2 faces are blue, 2 faces are red and 2 faces are green.

First we define the cycle index of the octahedron:

```
> psi := (x1, x2, x3, x4, x5, x6) -> (x1^6 + 6*x1^2*x4 + 3*x1^2*x2^2 + 6*x2^3 + 8*x3^2) / 24;
```

$$\psi := (x_1, x_2, x_3, x_4, x_5, x_6) \rightarrow \frac{1}{24} x_1^6 + \frac{1}{4} x_1^2 x_4 + \frac{1}{8} x_1^2 x_2^2 + \frac{1}{4} x_2^3 + \frac{1}{3} x_3^2$$

Then we define the variable substitution....

```
> s := k -> g^k + r^k + b^k;
```

$$s := k \rightarrow g^k + r^k + b^k$$

...and perform it on the cycle index:

```
> p := psi(s(1), s(2), s(3), s(4), s(5), s(6));
```

$$p := \frac{1}{24} (g+r+b)^6 + \frac{1}{4} (g+r+b)^2 (g^4+r^4+b^4) + \frac{1}{8} (g+r+b)^2 (g^2+r^2+b^2)^2 + \frac{1}{4} (g^2+r^2+b^2)^3 + \frac{1}{3} (g^3+r^3+b^3)^2$$

Finally we ask for the coefficient of $g^2 r^2 b^2$:

```
> coeff(coeff(coeff(p, g^2), b^2), r^2);
```

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> expand(p);
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$$2 g r b^4 + 2 g r^4 b + g^6 + r^6 + b^6 + g r^5 + g b^5 + 2 g^2 r^4 + 2 g^2 b^4 + g^5 r + g^5 b + 2 g^4 r^2 + 2 g^4 b^2 + 2 g^3 r^3 + 2 g^3 b^3 + r b^5 + 2 r^2 b^4 + r^5 b + 2 r^4 b^2 + 2 r^3 b^3 + 2 g^4 r b + 3 g^3 r^2 b + 3 g^3 r b^2 + 3 g^2 r^3 b + 6 g^2 r^2 b^2 + 3 g^2 r b^3 + 3 g r^3 b^2 + 3 g r^2 b^3$$