MAT 149A

## Homework 3

## due Wednesday January 29 in class

## 1. Biggs 5.6 # 19 page 112

**2.** Is there a permutation of  $\mathbb{N}_7$  which has order 10? Is there one of order 11? of order 9?

**3.** Let  $\alpha = (135)(24)$ . Find at least 6 permutations of  $\mathbb{N}_5$  that commute with  $\alpha$  (we say permutations  $\alpha$  and  $\beta$  commute if  $\alpha\beta = \beta\alpha$ ).

**4.** (a) How can you decide whether a permutation is even or odd if you know the lengths of its cycles?

(b) Prove that any permutation in  $S_n$  can be written as the product of at most n-1 transpositions.

5. Suppose you have an unlimited supply of water, a drain, a large container and two jugs which hold 7 and 9 liters, respectively. How would you arrange to put one liter of water in the container?

6. Biggs 6.6 # 5 page 129

What is the last digit in the base 10 representation of  $7^{93}$ ?