MAT 150B University of California Winter 2014

Homework 8

due March 14, 2014 in class

1. Prove or disprove: If an ideal I contains a unit, then it is the unit ideal.

2. (Artin 11.2.1) For which integers n does x^2+x+1 divide $x^4+3x^3+x^2+6x+10$ in $(\mathbb{Z}/n\mathbb{Z})[x]$?.

3. Prove that in the ring $\mathbb{Z}[x]$, $(2)\cap(x)=(2x)$.

4. Is the set of polynomials $a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$ such that 2^{k+1} divides a_k an ideal in $\mathbb{Z}[x]$?

5. (Artin 11.3.2) Prove that a nonzero ideal $I\subseteq\mathbb{Z}[i]$ contains a nonzero integer.

6. (Artin 11.3.3 b) Find generators for the kernel of the map $\mathbb{R}[x]\to\mathbb{C}$ defined by $f(x)\mapsto f(2+i)$.

7. (Artin 11.4.3 a and b) Describe each of the following rings:

(a) $\mathbb{Z}[x]/(x^2-3,2x+4)$.

(b) $\mathbb{Z}[i]/(2+i)$.