Homework 6
due February 27, 2015 for presentation in class

1. Define the lexicographic order on tabloids as follows. Associate with any $\{t\}$ the composition $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, where λ_i is the number of the row containing $n - i + 1$. If $\{s\}$ and $\{t\}$ have associated compositions λ and μ, respectively, then $\{s\} \leq \{t\}$ in lexicographic order if $\lambda \leq \mu$.

(a) Show that $\{s\} \trianglelefteq \{t\}$ implies $\{s\} \leq \{t\}$.

(b) Characterize the fact that $\{s\}$ is covered by $\{t\}$ in lexicographic order.

2. Consider the simple transpositions $s_k = (k, k+1)$ for $k \in \{1, 2, \ldots, n-1\}$ in S_n.

(a) Prove that the s_k generate S_n subject to the Coxeter relations

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\begin{aligned}
s_k^2 &= 1 & 1 \leq k < n, \\
s_k s_{k+1} s_k &= s_{k+1} s_k s_{k+1} & 1 \leq k \leq n - 2, \\
s_k s_\ell &= s_\ell s_k & 1 \leq k, \ell \leq n - 1 \text{ and } |k - \ell| \geq 2.
\end{aligned}
$$

(b) Show that if G_n is a group generated by g_k for $k = 1, 2, \ldots, n - 1$ subject to the relations above (replacing s_k by g_k), then $G_n \cong S_n$.
  Hint: Induct on n using cosets of the subgroup generated by $g_1, \ldots, g_{n-2}$.