Math 108 Final, Spring 2002

1) Show that the set {–6, –3, 0, 3, 6, ...} is denumerable. Use the definition of equivalent and show the required facts. Clearly state your function. After you create a function, there are 3 things you must show. (8 points)

2) Let $C = \{10, 11, 12, 13\}$. How many different relations can possibly be defined on C? Justify your answer. (3 points)

3) Prove that (a, ∞) is an open set, where a is any real number. (Note: $(a, \infty) = \{x \in R : a < x\}$.) (5 points)

4) Let A be a set. Recall a partition $\mathcal{A}$ of A satisfies

i) If $X \in \mathcal{A}$, then $X \neq \emptyset$.

ii) If $X \in \mathcal{A}$ and $Y \in \mathcal{A}$, then $X = Y$ or $X \cap Y = \emptyset$.

iii) $\bigcup_{X \in \mathcal{A}} X = A$.

Let R be an equivalence relation on A. Prove that the set of equivalence classes $\{a / R, a \in A\}$ is a partition of A. (Note: $a / \mathrm{R} = \{b \in A : a \,\mathrm{R}\, b\}$.) (8 points)

5) Use properties of an ordered field to show that

a) $0 < 1$ (2 points)

b) If $x < y$, then $-y < -x$. (2 points)

6) Prove that if $f : A \xrightarrow{1-1} B$, and $g : B \xrightarrow{1-1} C$, then $g \circ f : A \to C$ is also 1–1. (4 points)

7) In Math 141, geometry, we prove that two statements, Hilbert's Parallel Postulate, and Euclid's Parallel Postulate, are equivalent. Both of these two statements are themselves implications. Write Hilbert as A –> B, and Euclid as C –> D.

To show, for example, that Hilbert implies Euclid, we would try to show

(A –> B) –> (C –> D).

But in fact, the proof goes more like

[C /\ (A –> B)] –> D.

Use a truth table to show that these two forms are equivalent, meaning that the desired statement can be shown using the second method. Hint: A and B always go together as A –> B, so just treat this implication as a single variable in the truth table. (8 points)

Points:

8) a) Give an example, if possible, of a collection of open sets whose intersection is not open. (State "not possible" if it's not possible.) (2 points)

b) Give an example, if possible, of a collection of closed sets whose intersection is not closed. (State "not possible" if it's not possible.) (2 points)

9) a) Write the definition of a compact set. (2 points)

b) State the Heine-Borel Theorem. (2 points)

c) State the Bolzano-Weierstrass Theorem. (Hint: There is something about an "accumulation point.") (2 points)

Points: