

Math 108 Final

- 1) Prove that if  $x$  is an integer, then  $x^2 + 2x - 9$  is not divisible by 4.
- 2) Show that  $C - (A \cup \bar{B}) = \overline{A \cup (\bar{B} \cup \bar{C})}$ , where  $A$ ,  $B$ , and  $C$  are sets.
- 3) a) What is the cardinality of the set of rational numbers on  $[1, 3]$ ? Explain.  
b) What is the cardinality of  $[1, 3]$ ? (The real numbers  $x$  such that  $1 \leq x \leq 3$ .) Explain.
- 4) Let  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{1, 2\}$ . How many functions are there from...
  - a)  $A$  to  $B$  that are one-to-one?
  - b)  $A$  to  $B$  that are onto?
  - c)  $A$  to  $B$ ?
  - d)  $B$  to  $A$  that are one-to-one?
  - e)  $B$  to  $A$  that are onto?
  - f)  $B$  to  $A$ ?Briefly justify each answer.
- 5) a) Use properties of an ordered field to show that  $0 < 1$   
b) Produce a multiplication table for the finite field with these 7 elements, 0, 1, 2, 3, 4, 5, 6.
- 6) Let  $\mathcal{A}$  be a set. Recall a partition  $\mathcal{A}$  of  $A$  satisfies
  - i) If  $X \in \mathcal{A}$ , then  $X \neq \emptyset$ .
  - ii) If  $X \in \mathcal{A}$  and  $Y \in \mathcal{A}$ , then  $X = Y$  or  $X \cap Y = \emptyset$ .
  - iii)  $\bigcup_{X \in \mathcal{A}} X = A$ .

Let  $R$  be an equivalence relation on  $A$ . Prove that the set of equivalence classes  $\{a/R, a \in A\}$  is a partition of  $A$ . (Note:  $a/R = \{b \in A : a R b\}$ .)

- 7) Prove that the intersection of a finite collection of open sets is open.
- 8) a) Write the definition of a compact set.  
b) State any two of the following three theorems:
  - i) Heine-Borel Theorem.
  - ii) Bolzano-Weierstrass Theorem.
  - iii) Bounded Monotone Sequence Theorem.

Points: