

Math 132a: Stochastic Processes
Final Exam
(3/22/03)

Write your name and student ID number in the upper right-hand corner of this sheet and write your initials on each page of your exam.

Each problem is worth the same number of points. You **must** justify or defend your answer for each problem.

1. Suppose that you flip a fair coin four times. Let A_1 be the event that the first coin flips 1 and 2 agree, let A_2 be the event that coin flips 2 and 3 agree, let A_3 be the event that coin flips 3 and 4 agree, and let A_4 be the event that coin flips 4 and 1 agree. Are these four events pairwise independent? Are they mutually independent?
2. In the computer game Minesweeper, you have a field of squares, some of which contain mines. When you step on a square which is not a mine, it tells you the number of neighboring mines. Suppose that at the beginning each square in the game has a $\frac{1}{4}$ chance of being a mine and that all of these probabilities are independent. Suppose that later in the game you have three squares A , B , and C . Suppose that the game position tells you that either A is a mine and both B and C are safe, or that A is safe and both B and C are mines. What is the chance at that point that A is a mine?
3. Let X be a continuous random variable with density function e^{-x} and let Y be a discrete random variable with mass function

$$P[Y = n] = (1 - e^{-1})e^{-n}$$

for $n \in \mathbb{Z}_{\geq 0}$. Assuming that X and Y are independent, compute the density function of $X + Y$.

4. A Poisson random variable X with mean a has mass function

$$P[X = n] = \frac{a^n}{e^a n!}$$

for $n \in \mathbb{Z}_{\geq 0}$. What is its variance?

5. Let Θ be a random variable chosen uniformly in $[0, \pi]$. Find the density function of $X = \sin \Theta$.
6. Let X be a random variable in one dimension. Express the third moment $\mu_3(X + X^2)$ of the random variable $Y = X + X^2$ in terms of the moments of X .
7. Chernoff's inequality for a random variable X states that for all $t \geq 0$ and all x ,

$$P[X \geq x] \leq e^{-xt} M(t) = E[e^{(X-x)t}].$$

It is a fact that the right side is concave up as a function of t , so that it has a unique minimum. Prove this fact when X is a discrete random variable taking finitely many values x_1, x_2, \dots, x_n .