MAT 16A (A001) NAME: _____

Final Exam

Problem 1. (*estimated time: 15mn*) (**20 points**)

Find the derivatives of the following functions.

1. $f_1(x) = x^3 - 2x^2 + x - 1$

2. $f_2(x) = \dfrac{x^3+1}{(x-1)^2}$

3. $f_3(x) = \sqrt{x^2 + x - 1}$

4. $f(x) = \dfrac{3}{2}\sqrt[3]{(x^2 - x - 1)^2}$

5. $f_5(x) = \cos(2x+1) - x\sin(-x+1)$

Problem 2. (*estimated time: 50 mn*) (**70 points**)
Analyze and **sketch** the graph of the following function

$$f(x) = \begin{cases} \frac{x^2-6x+12}{x-4} & \text{if } x > 4 \\ \\ 3x^{2/3} + 3x^{1/3} & \text{if } x \leq 4. \end{cases}$$

Problem 3. (*estimated time: 15 mn*) (**20 points**)

Sketch a graph of **a continuous function** f having the following characteristics.

$f(-2) = f(6) = 0, \quad \lim_{x\mapsto-\infty} f(x) = -2 \quad \text{and} \quad \lim_{x\mapsto+\infty} f(x) = -4$

$f'(x) > 0$ if $x < 2, \quad f'(x) < 0$ if $x > 2, \quad f'(2)$ undefined and $f'(4) = -1$

$f''(x) < 0$ if $2 < x < 4, \quad f''(x) > 0$ if $-\infty < x < 2$ and $4 < x < \infty$

Problem 4. (*estimated time: 30mn*) (**30 points**)

Consider a semicircle of radius r with diameter AB (see figure).

(a) Show that for any point C in the semicircle, the triangle ABC is a right triangle at C. (This question is an **extra credit question**).

(b) Find the dimensions of the right triangle of largest perimeter (with hypothenuse AB) that can be inscribed in a semicircle of radius r.

(c) Show that this right triangle has also the largest area among all right triangles (with hypothenuse AB) that can be inscribed in a semicircle of radius r.

The problem 5 is a **extra credit problem**.

Problem 5. (*estimated time: 10mn*) (**20 points**)

(a) Why Parabolas didn't have inflection points?

(b) Give an example of a *continuous function* whose graph crosses its horizontal asymptote.