

1. Find the general solution of the differential equation

$$x^2y' - 2xy = 9.$$

2. The rate of increase in sales S (in thousands of units) of palm pilots is proportional to the current level of sales and inversely proportional to the square of the time t . That is,

$$\frac{dS}{dt} = \frac{kS}{t^2}$$

where k is a constant.

- (a) Is the constant, k , positive or negative?
(b) The saturation point for the market is 400,000 palm pilots. That is, the limit of S as $t \rightarrow \infty$ is 400. After 1 year, 100,000 palm pilots have been sold. Find S as a function of time in years.
3. Sketch the region of integration and evaluate the following integral.

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{1}{x^3 + 2} dx dy$$

4. Find the critical points of the function $f(x, y) = x + y + 1/(xy)$ and determine whether they are relative minima, relative maxima, or saddle points.
5. Use Lagrange multipliers to find the point on the plane $x + 2y + 3z = 14$ which is closest to the origin. (*Hint*: Minimize the square of the distance in order to avoid square roots.)
6. Find the first 5 terms of the sequence of partial sums associated to the series

$$\sum_{n=0}^{\infty} (-1)^n 2^n$$

7. Determine whether the following series converge or diverge. Justify your answers.

(a)
$$\sum_{n=1}^{\infty} \frac{3n^3 + 1}{2n^3 + n^2 - 5}$$

(b)
$$\sum_{n=1}^{\infty} \pi n^{-\frac{5}{2}}$$

(c)
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{4^n}$$

8. Determine whether the following geometric series converge. If they do, find their sum.

(a)
$$\sum_{n=0}^{\infty} \left(-\frac{\pi}{2}\right)^n$$

(b)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{n+1}}{5^{n-1}}$$

9. Find the radius of convergence of the following power series.

(a)
$$\sum_{n=0}^{\infty} \frac{n^2(x+7)^n}{n!}$$

(b)
$$\sum_{n=0}^{\infty} \frac{(3x)^n}{2^n}$$

10. (a) Find the 3rd Taylor polynomial centered at 0 of

$$f(x) = \frac{1}{\sqrt[3]{1-x}}.$$

(b) Find the 6th Taylor polynomial centered at 0 of

$$f(x) = \frac{1}{\sqrt[3]{1-x^2}}$$

Extra credit: Suppose that a rubber ball, when dropped on a concrete patio, rebounds 90 percent of the distance it falls. Find the total distance, both up and down, traveled by the ball if it is dropped from a height of 6 feet.