

Math 16C Final, Spring 2002

1) For each series,

i) Determine if it converges.

ii) Justify your reasoning.

iii) Find the sum, if possible.

(3 points each)

a) $\sum_{n=0}^{\infty} \frac{(2.1)^n}{n!}$

b) $\sum_{n=1}^{\infty} (0.4)^n$

c) $\sum_{n=1}^{\infty} \frac{1}{n(\sqrt[4]{n})}$

d) $\sum_{n=1}^{\infty} \frac{n-1}{n}$

2) Find the Taylor series for $f(x) = (3x + 7)^{-2}$ centered at $c = -1$ by following steps a–d.

a) Find the first three derivatives of f . (3 points)

b) Evaluate $f(-1)$, $f'(-1)$, $f''(-1)$, and $f'''(-1)$. (2 points)

c) Find a formula for $f^{(n)}(-1)$. (2 points)

d) Use the Taylor's Theorem to write the answer. (2 points)

3) The function $g(x, y) = 3x^2y - 4xy + y^2$ has three critical points. Find them, and classify each one. (Classify means determine what type of critical point each one is.) (11 points)

4) Newton's method. Consider $h(x) = x^4 - x - 1$. Using derivatives and related analysis, it can be proven that this function has exactly two roots. (A root is a value x where $h(x) = 0$.)

a) Determine the two unit intervals that contain roots. A unit interval is an interval from one integer to the next, like the interval $[3, 4]$. Explain your reasoning. (5 points)

b) For the larger root, make an initial guess x_1 . Use Newton's Method to find x_2 . Then write a formula for x_3 , but do not simplify. (5 points)

5) Explain how you would estimate $\cos(0.5)$ to within 0.001 using a calculator with no trig functions. (4 points)

6) Solve the differential equation $\frac{dy}{dx} = \frac{x+1}{e^y}$, subject to the condition $y(2) = \ln 8$. Solve your answer for y , a function of x . (4 points)

Total points for this page:

Extra Credit. Only 4 points, but give it a try if you want an A+:

You are told that $g(x, y)$ is a function of two variables, and you are given the two partial derivatives:

$$g_x = \sin y + y^2(e^x) - 1$$

$$g_y = x(\cos y) + 2y(e^x) + 2y.$$

Find all possible functions g . Hint: do something like antidifferentiation, but keep in mind that these are functions of two variables.

Total points for this page: