

1. [39 pts.] Evaluate the following integrals.

(a)  $\int \frac{e^x - 1}{e^{2x} + 1} dx$

(b)  $\int \frac{\cos \theta}{2 + \sin^2 \theta} d\theta$

(c)  $\int \frac{dx}{(\sqrt{x^2 + 1})^3}$

2. [10 pts.] The Laplace transform of a function  $f(t)$  is defined by

$$L(s) = \int_0^{\infty} e^{-st} f(t) dt$$

where  $s$  is a parameter. Compute the Laplace transform of  $f(t) = t$ .

3. [12 pts.] Find the derivative of  $G(x) = \int_{\sin 3x}^{\cos 3x} 2^t dt$ .

4. [12 pts each]

(a) Integrate the improper integral  $\int_0^9 \frac{1}{\sqrt[3]{(x-8)^4}} dx$ .

(b) Determine whether  $\int_0^{\infty} e^{-x} \sin x^5 dx$  converges or diverges. **Do not evaluate.**

5. Let  $R$  be the region bounded by  $y = x^3 + 2x + 1$ ,  $y = 1$ ,  $x = 0$ , and  $x = 1$ . **Set up, but do not evaluate** an integral(or integrals) for:

(a) [7 pts.] the volume of the solid generated by revolving  $R$  about the  $x$ -axis.

(b) [7 pts.] the volume of the solid generated by revolving  $R$  about  $x = 3$ .

(c) [12 pts.]  $\bar{x}$ , the  $x$ -coordinate of the centroid of  $R$ .

6a) [10 pts] Sketch the graph of  $r = \sin 2\theta$ .

6b) [13 pts.] Find the area of one of the loops.

7. A curtate cycloid, is given by the parametric equations

$$x = t - \sin t \quad y = 1 - \cos t$$

(a) [10 pts.] Find the value of  $t$ ,  $0 \leq t \leq 2\pi$ , at which the graph has a maximum(that is, find where the slope of the tangent line is zero).

(b) [5 pts.] Find  $\frac{d^2y}{dx^2}$ .

8. [15 pts.] Suppose that the growth of an experimental population of rabbits is directly proportional to the number of rabbits in the experiment at any given time. If there are 100 rabbits after 2 months of the experiment and 300 rabbits after 4 months, how many rabbits were in the original population.

9. [5 pts. each] Let  $f(x) = x^{2/3}$ .

(a) Find the average value of  $f(x)$  over  $[1, 8]$ .

(b) State the mean value theorem for definite integrals.

(c) Find the value of  $x$  at which  $f(x)$  attains its average value over  $[1, 8]$ .

10. [5 pts. each]

(a) State the limit definition of the definite integral.

(b) [5 pts.] For which type of functions does this limit always exist?

11. Let  $f(x) = \frac{1}{1+x^2}$  on  $[0, 2]$ . Partition  $[0, 2]$  into 4 sections of equal length.

(a) [10 pts.] Form a Riemann sum using right endpoints as sampling points. Do not compute this sum.

(b) [5 pts.] Should the estimate in (a) be greater than or less than the value of

$\int_0^2 \frac{1}{1+x^2} dx$ ? Clearly explain why.