Math 21B Final

1) Which two mathematicians are said to have invented calculus? (2 points)

2) What is a "mesh"? What "happens" to it when we find a definite integral? Why? (6 points)

3) State the Mean Value Theorem for Definite Integrals. (6 points)

4) What is an "elementary" function? What kinds of functions are not elementary? (4 points)

5) Find $\int x^2 e^{4x}\,dx$ (6 points)

6) Find $\int \frac{4x^2}{3+8x^3}\,dx$ (6 points)

7) Let $F(x) = \int_0^x e^{-t^2}\,dt$. Is $F(x)$ concave up or concave down at $x = 2$? Justify. (4 points)

8) Find the centroid of this triangle by following these four steps.

a) Find equations for the two diagonal lines. (4 points)

b) Find the moment about the y-axis by starting with integrals like (6 points)

$$\int_0^b \ldots + \int_b^a \ldots$$

c) Find the area of the triangle. (4 points)

d) Find $\bar{x}$. (4 points)

9) Find $\int \frac{1}{\sqrt{9+x^2}}\,dx$. (15 points)

Here are some integrals that might be of assistance:

$$\int \sec x\,dx = \ln|\sec x + \tan x|$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x|$$

$$\int \tan x\,dx = \ln|\sec x|$$

$$\int \tan^2 x\,dx = \tan x - x$$

$$\int \tan^3 x\,dx = \frac{1}{2}\tan^2 x + \ln|\cos x|$$

10) a) Graph $r = 3 + 2 \cos \theta$. (5 points)
b) Find the area of the region inside this curve. (5 points)

11) Graph $r = 5 \csc \theta$. (5 points)

12) Draw a picture and write 2 or 3 equations to justify this formula for arc length in polar coordinates:

$$ds = \sqrt{r^2 + (r')^2}\, d\theta$$ (5 points)

13) a) Determine whether $\int_1^\infty \frac{3}{x^{1.3}} dx$ converges. (4 points)

b) Determine whether $\int_1^\infty \frac{3}{x^{1.3} + x} dx$ converges. (3 points)

14) a) Is $\int_{-3}^{3} \frac{1}{x^2 - 3x - 10} dx$ an improper integral? Explain. (3 points)

b) Is $\int_{-3}^{3} \frac{1}{x^2 - 3x + 10} dx$ an improper integral? Explain. (3 points)

Extra Credit: An object is launched from the origin at an angle α above the x-axis. The initial speed is v_0. The x and y positions as functions of time are as follows:

$$x(t) = v_0 (\cos \alpha)\, t$$
$$y(t) = v_0 (\sin \alpha) t - 16t^2.$$

Eliminate t to find the equation of the curve travelled as a function of x. Hint: to simplify the algebra, let

$$A = v_0 (\cos \alpha)$$
$$B = v_0 (\sin \alpha).$$

What kind of shape is this?