Final 21C Winter 2002

1. (9) Compute f_y where $f(x,y) = xy^2 e^{x^3 \sin(\pi y)}$.

2. a. (8) Suppose you know at some point that

$$\frac{\partial f}{\partial x} = 2, \quad \frac{\partial f}{\partial y} = -3, \quad \frac{\partial x}{\partial s} = -5, \quad \frac{\partial x}{\partial t} = 5, \quad \frac{\partial y}{\partial s} = 4, \quad \frac{\partial y}{\partial t} = -2.$$

What is the value of $\frac{\partial f}{\partial t}$?

b. (8) Suppose in addition that you know $\frac{\partial s}{\partial u} = 2$, $\frac{\partial t}{\partial u} = 3$, what is the value of $\frac{\partial f}{\partial u}$?

3. (13) Change the order of integration for the following integral and <u>evaluate</u>.

$$\int_0^2 \int_{2x}^4 \frac{1}{1+y^2} \, dy \, dx.$$

4. (12) Write the following integral in spherical coordinates. <u>DO NOT EVALUATE</u>

$$\int_0^{2\pi} \int_0^1 \int_{r^2}^{r} r^2 \, dz \, dr \, d\theta.$$

5. (12) Find the critical points of $f(x,y) = 6xy^2 - 2x^3 - 3y^4 + 1$

6. (13) Given that (0,0) and (0,4) are critical points for $z = 4y^3 + 12yx^2 - 24x^2 - 24y^2$,

determine whether each is a local min, max, or saddle.

7. (25) Determine whether the following series converge absolutely, conditionally, or diverge. You must state the test used.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n^3}$

b. $\displaystyle\sum_{n=1}^{\infty} n e^{-n^2}$

c. $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{\pi}{n^2}\right)$

8. (13) Find the Maclaurin series for $f(x) = (5 + 2x)^{-1}$ using the Maclaurin formula for the terms a_n

9. (12) For the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n \cdot 3^n}$

a. What is the radius of convergence?

b. Where does the series converge absolutely?

c. Where does the series converge conditionally?

10. (10) Use Maclaurin series to compute $\displaystyle\lim_{x \to 0} \frac{1-\cos(x^2)}{x^4}$

11. (15) a. Use the principles developed in this course to estimate $\displaystyle\int_0^{1/2} \frac{1}{1+x^3}\,dx$.
You don't have to complete any computations that would normally be done on a calculator.

b. Estimate the size of the error that results from doing your estimate in part a.