

1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 8 \\ 2 & 3 & 7 \end{bmatrix}$

(a) [10 pts.] Find $\det(A)$.

(b) [4 pts.] how many solutions does $A\vec{x} = \vec{0}$ have? Why?

2. [12 pts.] Solve the following system of equations **using the inverse**.

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ -2x_1 + 3x_2 &= 1 \end{aligned}$$

3. [14 pts.] What are all the subspaces of \mathbb{R}^3 ? What is the dimension of each subspace?

4. Let $A = \begin{bmatrix} 2 & 2 & -1 & -4 \\ 1 & 1 & 1 & -5 \\ 3 & 3 & -1 & -7 \end{bmatrix}$

(a) [12 pts.] Show that A can be reduced to the following reduced row echelon form:

$$A = \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) [10 pts.] Find a basis for the nullspace of A .

(b) [8 pts.] Find a basis for the row space of A .

(c) [8 pts.] Find a basis for the column space of A .

(d) [6 pts.] What is the nullity of A ? What is the rank of A ? 5. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$.

(a) [12 pts.] Find the eigenvalues and eigenvectors of A .

(b) [8 pts.] Find the general solution to the system of differential equations

$$\vec{x}'(t) = A\vec{x}(t).$$

6. [16 pts.] Let A be an $n \times n$ matrix. Let λ be a particular eigenvalue of A . Show that the set S consisting of all eigenvectors of A associated with λ , as well as the zero vector, is a subspace of \mathbb{R}^n .

7a. [8 pts.] What is the definition of vectors $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ being linearly independent.

7b. [8 pts.] Let V be a vector space. Define what it means for W to be a subspace of V . Be specific.

8. [14 pts.] Show that if A is similar to B (i.e. there exists P such that $A = P^{-1}BP$) then $\det(A) = \det(B)$.

9. Let $L : P_2 \rightarrow \mathbb{R}^2$ be the map defined by $L(at^2 + bt + c) = (a, b)$.

(a) [14 pts.] Show L is a linear transformation.

(b) [10 pts.] Find a basis for the $\ker L$.

(c) [10 pts.] Find a basis for the range L .

(d) [10 pts.] Is L one-to-one? Is L onto?

(e) [6 pts.] Does L have an inverse?