Math 22A–003 Spring 2002
Final Exam

1) (8 pts) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$.

a) Find A^{-1}.

b) Use your answer in (a) to solve $Ax = b$.

2) (8 pts) Suppose the LU factorization of a matrix A is given by $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. Use this to solve the equation $Ax = b$, where $b = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

3) (10 pts) Let P_2 be the vector space of polynomials of degree 2 or smaller (at^2+bt+c), and P_1 be the space of polynomials of degree 1 or smaller $(at + b)$.
Let $L : P_2 \to P_1$ be the linear transformation defined by: $L(p(t)) = p'(t)$.

a) Find a basis for $\ker L$.

b) Find a basis for range L.

c) Is L one-to-one? Justify your answer.

4) (8 pts) Prove: If A is a nonsingular matrix with eigenvalue λ and associated eigenvector x, then A^{-1} has eigenvalue $\frac{1}{\lambda}$ with associated eigenvector x.

5) (10 pts) Let $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 1 & 5 \\ -1 & -2 & -5 & 2 \end{bmatrix}$.

a) Find a basis for the null space of A.

b) Find a basis for the column space of A.

c) Find a basis for the row space of A.

d) What is rank A?

6) (6 pts) Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -3 & 5 \\ 4 & 4 & -9 \end{bmatrix}$. Find $\det(A)$.

7) (8 pts) Prove: If $Ax = b$ has more than one solution, then it has infinitely many

solutions.

8) (8pts) Let $S = \{v_1, v_2, v_3\}$, where $v_1 = (1, -1, 2)$, $v_2 = (2, -1, 2)$, and $v_3 = (-4, 1, -2)$. Find a basis for span S.

9) (8pts)

a) Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. That is, find a matrix P and a diagonal matrix D so that $D = P^{-1}AP$. (You do not need to compute P^{-1}.)

b) Is P an orthogonal matrix? Justify your answer.

DO ONLY ONE OF THE TWO VERSIONS OF PROBLEM 10.
CLEARLY INDICATE WHICH PROBLEM YOU WANT GRADED.

10) (8 pts) Let $f(x) = e^{2x}$ and $g(x) = 3x$. Using the usual inner product on the interval $[0, 1]$, find:

a) (f, g)

b) $\|f(x)\|$

10) (8 pts) Let $w_1 = (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$ and $w_2 = (\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3})$, and let $W = \text{span}\{w_1, w_2\}$.

a) Show that w_1 and w_2 are orthonormal.

b) Let $v = (1, 2, 3)$. Find $\text{proj}_W v$.

11) (8 pts) Prove: If $S = \{u_1, u_2, \ldots, u_k\}$ is an orthogonal set of nonzero vectors in R^n, then S is linearly independent.

12) (10 pts) Decide whether each of the following statements is true or false (circle T or F).

a)	T	F	$Ax \cdot y = x \cdot A^T y$ for any $n \times n$ matrix A and n-vectors x and y.
b)	T	F	If L:V$\rightarrow$W is linear, then dim(ker L) + dim(range L) = dim(V).
c)	T	F	A given vector space may have more that one zero vector.
d)	T	F	Let A be a 4×4 matrix. If there are 2 parameters in the solution of $Ax = 0$, then A has 3 linearly independent columns.
e)	T	F	Let T be the transition matrix of a regular Markov process, and let u be the steady–state vector ($Tu = u$). Then for any initial state vector x, the process eventually approaches the steady–state vector ($T^n x \rightarrow u$ as $n \rightarrow \infty$).