NAME:_____

SID:_____

MAT22B–1: Final Exam

(Instructor: Lieu)

Monday, December 8, 2008

INSTRUCTIONS:

1. Show work for partial credit.
2. Clearly **label** each part (a),(b),(c), etc.
3. **Box** your answer, no box no full credit.
4. No calculators or notes allowed.

No.	Score	
1		20
2		20
3		20
4		25
5		25
6		35
7		55

Total:_____/200

Table 1. Elementary Laplace Transforms

1	1	$\frac{1}{s}, \quad s > 0$
2	e^{at}	$\frac{1}{s-a}, \quad s > a$
3	$t^n,\ n > 0$ integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4	$\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
5	$\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
6	$\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
7	$\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
8	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
9	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
10	$t^n e^{at},\ n > 0$ integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
11	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
12	$(-t)^n f(t)$	$F^{(n)}(s)$

Table 2. Simple Trigonometric Integrals

1	$\int \sec^2 x \ dx = \tan x + C$
2	$\int \csc^2 x \ dx = -\cot x + C$
3	$\int \sec x \tan x \ dx = \sec x + C$
4	$\int \csc x \cot x \ dx = -\csc x + C$
5	$\int \sec x \ dx = \ln\lvert\sec x + \tan x\rvert + C$
6	$\int \csc x \ dx = -\ln\lvert\csc x + \cot x\rvert + C$
7	$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$
8	$\int \frac{dx}{1+x^2} = \arctan x + C$

Table 3. Some Trigonometric Identities

1	$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$
2	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
3	$\sin A \pm \sin B = 2\sin\left(\frac{A\pm B}{2}\right)\cos\left(\frac{A\mp B}{2}\right)$
4	$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
5	$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

1. (20pts) Solve the initial valued problem $y' = 2y^2 + xy^2,\ y(0) = 1$, and determine the interval of definition.

2. (20pts) Solve the initial valued problem

$$y^{(4)} - y = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1$$

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3. (20pts) Find the general solution of $y'' - 2y' + y = e^t$.

4. (25pts) Consider the Critical Threshold model for population growth

$$\frac{dy}{dt} = -r\left(1 - \frac{y}{T}\right) y,$$

where r and T are given positive constants.

(a) (5pts) Find all critical (equilibrium) points and determine whether each is asymptotically stable, unstable, or semistable.

(b) (15pts) Sketch the graph of several solutions in the ty-plane, including all equilibrium solutions.

(c) (5pts) Describe the asymptotic behavior ($t \to \infty$) of the solution $y_1(t)$ with the initial condition $y_1(0) = b$ where $0 < b < T/2$.

5. (25pts) Suppose the initial value problem describing the motion of a damped unforced vibrating system is

$$u'' + u' + u = 0, \quad u(0) = 1, \quad u'(0) = -2.$$

(a) (15pts) Solve the initial value problem to find the position of the mass at any time.

(b) (10pts) Sketch the graph of the solution. (Hint: Transform to the form $Re^{at}\cos(w_0 t + \delta)$.)

6. (35pts) Find the general solution and draw the phase portrait for the following system of ODE's

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

7. (55pts) A hard-boiled egg is removed from the boiling water and placed out in the open to cool. Assume that the temperature of the room is controlled at 25^oC, and the initial temperatures of the yolk and the whites are 45^oC and 55^oC, respectively. Suppose $\alpha = 2$ is the conductivity constant of the membrane between the yolk and the whites and $\beta = 3$ is the conductivity constant of the shell. Let T_1 and T_2 denote the temperature of the yolk and whites, respectively.

   (a) (10pts) Apply Newton's Law of Cooling to set up the system of ODE's.

   (b) (45pts) Find the temperatures T_1 and T_2 as functions of time t.

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