MAT22B-01 Final Fall 1999

1. [10 pts.] Consider the DE:

$$y''' + p(t)y'' + q(t)y' + s(t)y = g(t) \qquad g(t) \neq 0$$

Circle all of the classifications that apply to this DE. This DE is:

Partial
Ordinary
Linear
Nonlinear
Homogeneous
Nonhomogeneous
Constant Coefficient
In Standard Form
Seperable
Exact
Y2K Compatible

2. [15 pts.] Using the method of undetermined coefficients, find the appropriate form for the particular solution of the following equation. Be precise as possible. **DO NOT solve for the constants!**

$$y'' - 3y' + 2y = 2t^2 + e^t + 2te^t + 4e^{3t} - \sin 2t$$

3. [15 pts.] Can the **functions** $y_1(t) = \cos^2 t$ and $y_2(t) = \sec t$ be a fundamental set of solutions to the ODE $y'' + p(x)y' + q(x)y = 0$ on the interval $(\frac{\pi}{2}, \frac{3\pi}{2})$?

4. [15 pts.] Solve the IVP:

$$\frac{dy}{dt} = -\frac{y \cos t}{1 + 2y^2} \qquad y(0) = 1$$

5. Consider the following ODE:

$$e^{2y} - y\cos(xy) + (2xe^{2y} - x\cos(xy) + 2y)y' = 0$$

(a) [8 pts.] Show that the ODE is exact.

(b) [7 pts.] What does the exactness of **this** ODE tell us? **DO NOT solve the ODE!**

6. [20 pts.] Solve the following ODE

$$ty'' - 2ty' + ty = e^t$$

7. [20 pts.] Using the Laplace Transforms:

$$L\{\sin at\} = \frac{a}{s^2 + a^2}$$

$$L\{\cos at\} = \frac{s}{s^2 + a^2}$$

And the equation:

$$\frac{s^2 + s + 4}{(s^2+4)(s^2+9)} = \frac{1}{5}\left(\frac{s}{s^2+4}\right) - \frac{1}{5}\left(\frac{s-5}{s^2+9}\right)$$

Solve $y'' + 9y = \cos 2t \quad y(0) = 0 \quad y'(0) = 1$.

8. [20 pts.] Show that if a and λ are positive constants, and b is any real number, then every solution to

$$y' + ay = be^{-\lambda t}$$

has the property that $y \to 0$ as $x \to \infty$.

Hint: Consider the case $a = \lambda$ and $a \neq \lambda$ seperately.

9. Consider the system of ODEs:

$$\vec{x}' = \begin{pmatrix} -5 & 3 \\ -3 & 1 \end{pmatrix} \vec{x}$$

(a) [15 pts.] Find the general solution of the system.

(b) [5 pts.] Sketch the phase portrait of the solutions.

10. [20 pts.] Solve the PDE:

$$\begin{aligned} u_{xx} &= 4u_{tt} \qquad 0 < x < 8 \qquad t > 0 \\ u(0,t) &= 0 \qquad\qquad u(8,t) = 0 \qquad t > 0 \\ u(x,0) &= 2\sin\frac{3\pi x}{4} - 3\sin 2\pi x \\ u_t(x,0) &= 4\pi \sin\frac{\pi x}{2} \end{aligned}$$

11. [30 pts.] **Derive** the General Solution to the following PDE:

$$\begin{aligned} tu_{xx} + u_t &= 0 \qquad 0 < x < l \qquad t > 0 \\ u(0,t) &= 0 \qquad\qquad u(l,t) = 0 \qquad t > 0 \\ u(x,0) &= f(x) \qquad 0 < x < l \end{aligned}$$