

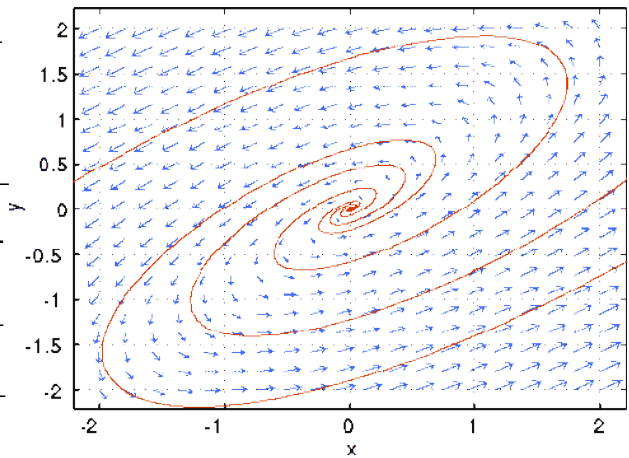
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Student ID# _____

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- 1) Please do not turn this page until instructed to do so.
- 2) Your name and signature will be compared to your white card. A student who's signature does not match the signature on his/her white card will receive a zero, and will be asked to come to office hours to discuss the issue before it is sent to Student Judicial Affairs.
- 3) Keep your SID or other photo ID out and in plain sight during the exam. In the spaces above, write the names of your left and right Neighbors on your exam (or wall if you are on the end).
- 4) It is a violation of the university honor code to, in any way, assist another person in the completion of this exam or to copy answers from someone else's exam. Please keep your own work covered up as much as possible so that others will not be tempted or distracted. Thank you for your cooperation.
- 5) No calculators, books, notes, phones, classmates, iPods, or other electronic devices, tools, or aides may be used for this exam. Just a pencil and paper. Additional scratch pages are on the back.
- 6) Read Directions carefully. Show all work unless told otherwise to receive full credit. A correct answer with no supporting work will not receive full credit. What you write down and how you write it are important means of getting a good score on this exam. Neatness and organization are also important.

<i>Problem</i>	<i>Points</i>	<i>Problem</i>	<i>Points</i>	<i>Problem</i>	<i>Points</i>
1 (30 points)		7 (10 points)		13 (20 points)	
2 (20 points)		8 (10 points)		14 (20 points)	
3 (20 points)		9 (10 points)		15 (20 points)	
4 (30 points)		10 (10 points)		16 (10 points)	
5 (10 points)		11 (30 points)		17 (10 points)	
6 (10 points)		12 (20 points)		18 (10 points)	

QUIZ 1

1) Find the general solution to the given Ordinary Differential Equations

a) $ty' + y = e^{2t}$

b) $y' - t^2 y^2 = 0$

c) $\frac{dy}{dx} = \frac{-4x - 5y}{5x + 6y}$

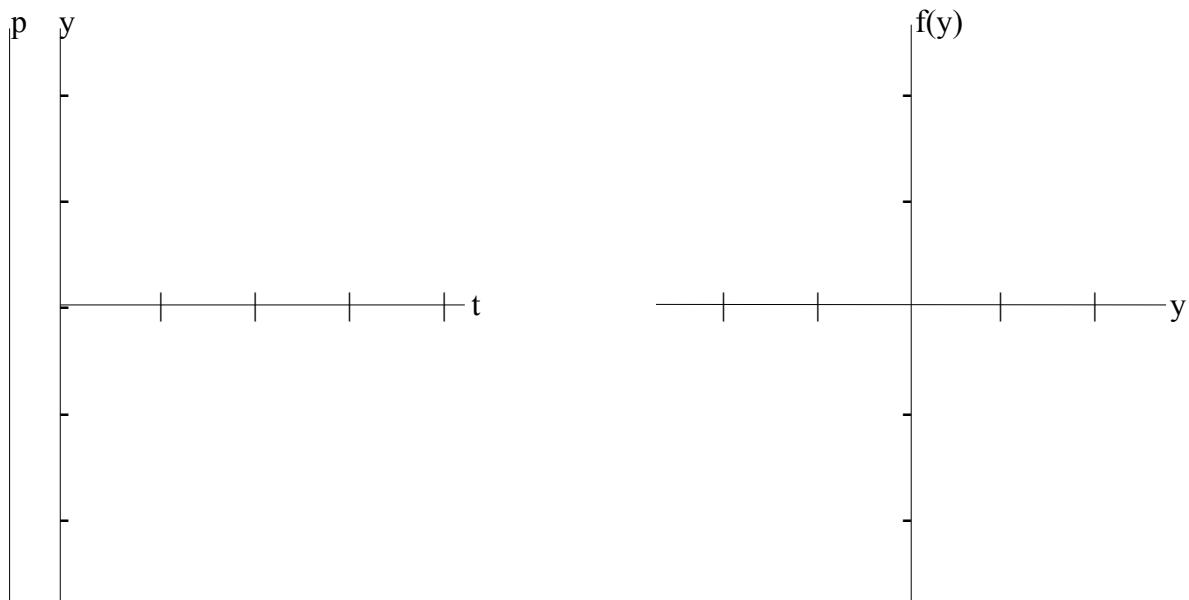
2) A tank originally contains 30 gallons of pure water. Then, water containing 2 grams of salt per gallon is poured into the tank at a rate of 3 gallons per minute. Water leaves the tank at the same rate.

a) Write out an IVP problem which when solved will tell you how many grams of salt are in the tank at any time t .

b) Find the amount of salt in the tank at any time t .

3) Let $\frac{dy}{dt} = f(y)$ where $f(y) = \sin|\pi y|$ on $-2.5 < y < 2.5$

a) Sketch the graph of $f(y)$ versus y , Draw the Phase Line (p), and sketch several graphs of solutions in the ty plane (at least 1 for each fundamentally different solution).



b) Find the equilibrium solutions and determine whether they are stable, unstable, or semi-stable.

QUIZ 2

4) Find the General Solution of the given Homogeneous Differential Equations.

a) $y'' - 20y' + 100y = 0$

b) $y'' - 5y' - 50y = 0$

c) $y'' + 10y' + 125y = 0$

5) Verify that $y_1(t) = \frac{e^t + e^{-t}}{2}$ and $y_2(t) = \frac{e^t - e^{-t}}{2}$ are linearly independent on any open interval I.

6) Solve the Initial Value Problem $L[y]=0$, $y(0)=0$, $y'(0)=0$. Justify your answer.

7) Given $t^2 y'' + 3t y' + y = 0$ has a solution $y_1(t) = \frac{1}{t}$, use the Reduction of Order technique to find the general solution.

8) Use the Method of Undetermined Coefficients to find the general solution to the following non-homogeneous equations.

$$y'' - 20y' + 100y = 400e^{-10t}$$

QUIZ 3

9) Find $\mathcal{L}\left\{ \frac{e^{-t} + e^t}{2} \right\}$

10) Given $\mathcal{L}\{ t^n e^{at} \} = \frac{n!}{(s-a)^{n+1}}$, use the Laplace Transform to solve for y.

$$y'' + 4y' + 4y = 0 \quad y(0) = 0, \quad y'(0) = -2$$

11) Consider two interconnected tanks which initially contain pure water, namely Tank 1 which has 30 gallons pure water and Tank 2 which has 20 gallons pure water. A salt mixture of 2 grams per gallon is poured into Tank 1 at a rate of 3 gallons per minute, and Tank 1 is pumping water into Tank 2 at a rate of 5 gallons per minute. A salt mixture of 1 gram per gallon is being pumped into Tank 2 at a rate of 1 gallon per minute, and Tank 2 is pumping water into Tank 1 at a rate of 2 gallons per minute, and water is exiting Tank 2 into the local river at a rate of 4 gallons per minute.

a) Draw a diagram to show what is happening in this waterworks system.

b) Create a system of differential equations to determine that amount of salt $Q_1(t)$ and $Q_2(t)$ in Tank 1 and Tank 2.

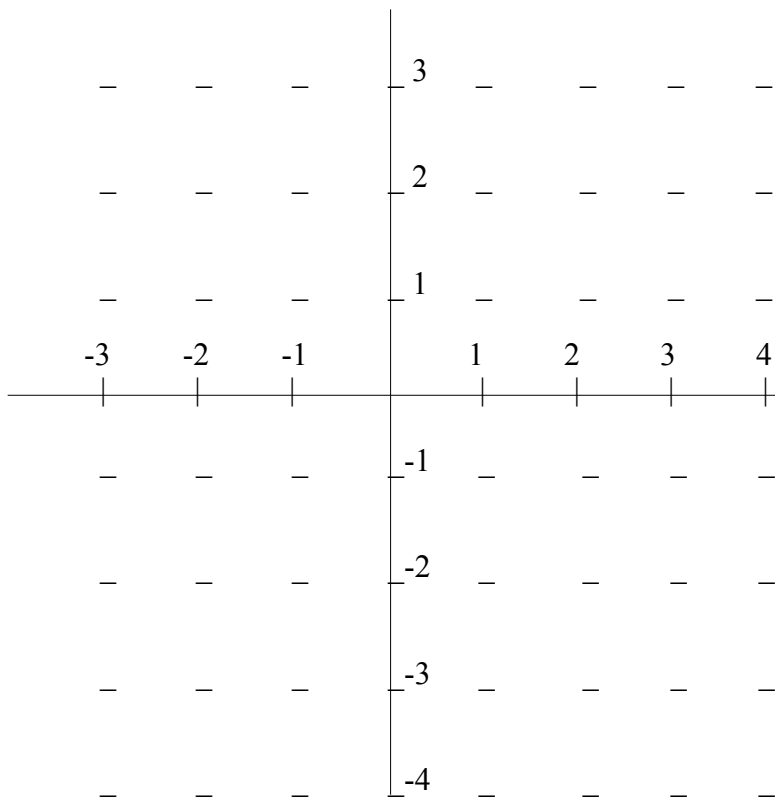
c) Transform the System of ODEs into a second order ODE by substituting the first equation into the second equation.

QUIZ 4

12) Consider the linear system $\dot{X} = \begin{pmatrix} -4 & 3 \\ 2 & -3 \end{pmatrix} X$

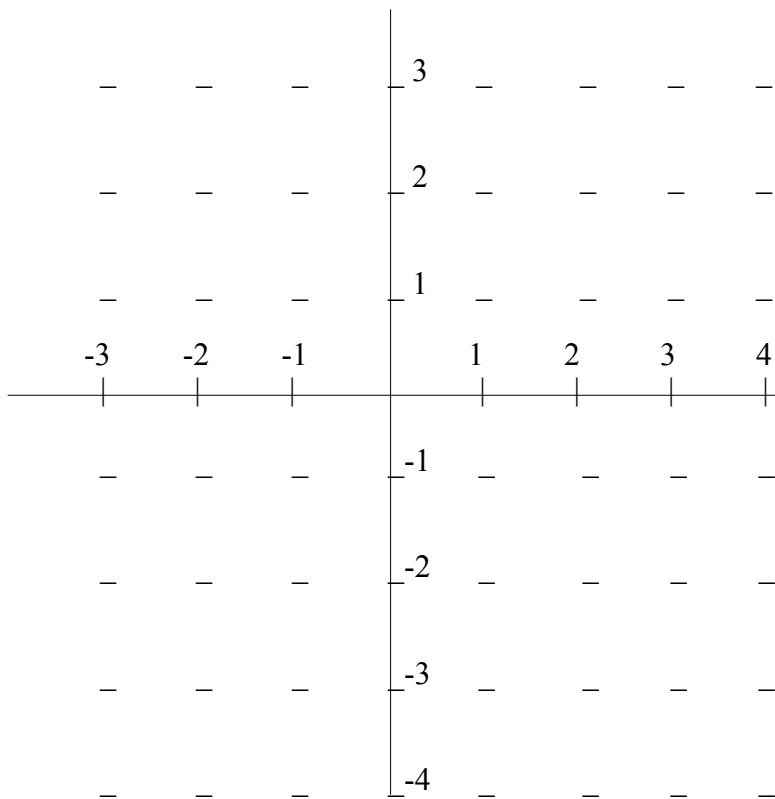
a) Find the general solution.

b) Use your solution to create a phase portrait for the system.



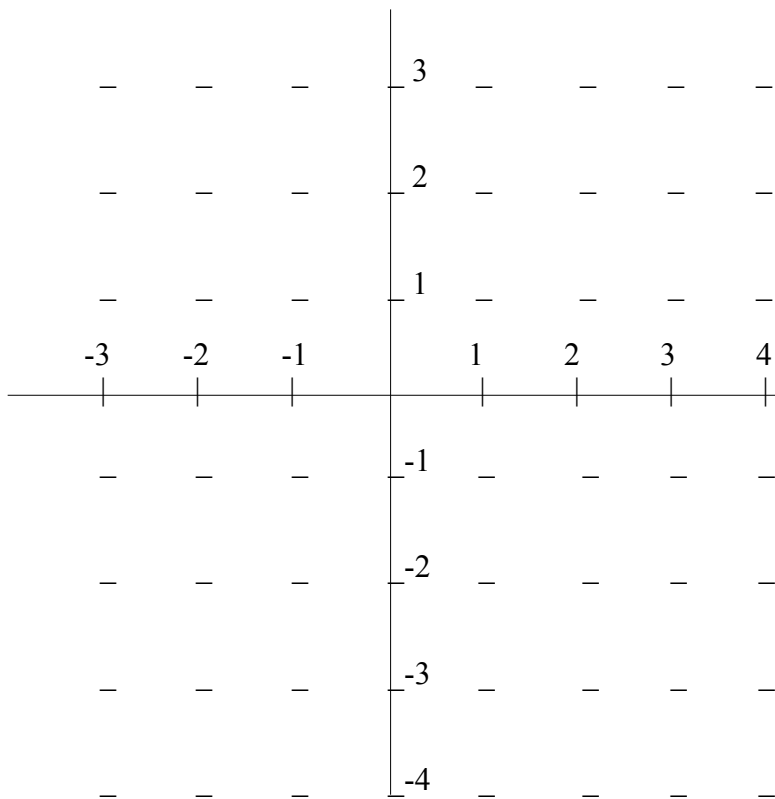
13) Consider the linear system $\dot{X} = \begin{pmatrix} 4 & 3 \\ -2 & -3 \end{pmatrix} X$
 a) Find the general solution.

b) Use your solution to create a phase portrait for the system.



14) Consider the linear system $\dot{X} = \begin{pmatrix} -5 & 10 \\ -10 & -5 \end{pmatrix} X$
 a) Find the general solution.

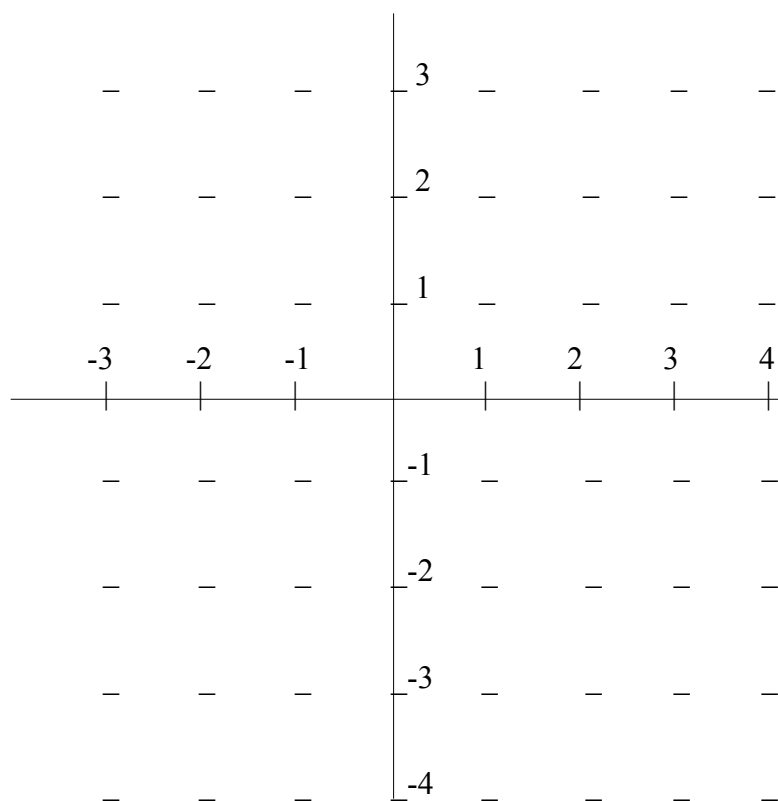
b) Use your solution to create a phase portrait for the system.



15) Consider the linear system $\dot{X} = \begin{pmatrix} -5 & -3 \\ 3 & 1 \end{pmatrix} X$

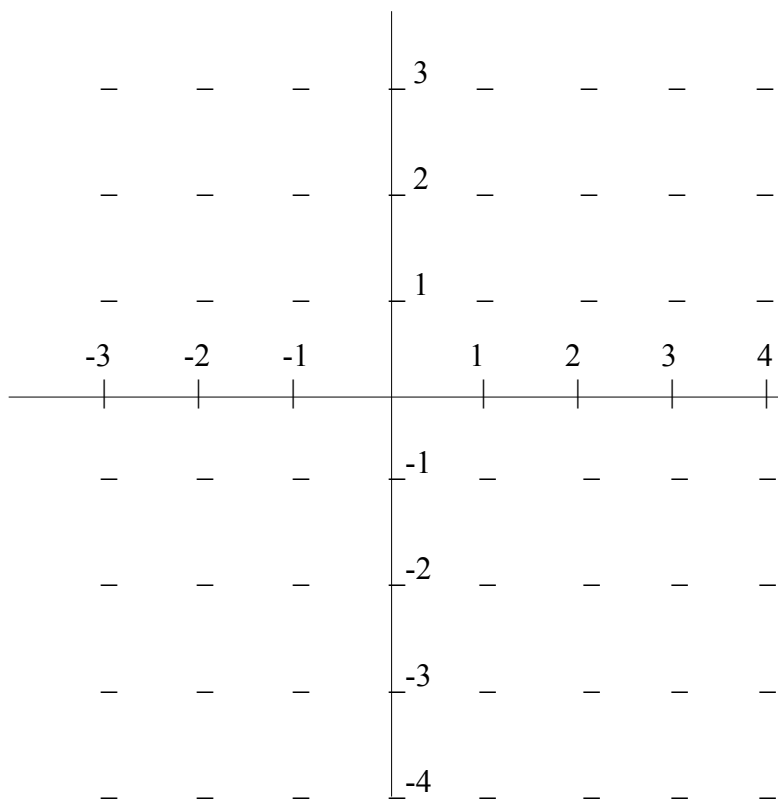
a) Find the general solution.

b) Use your solution to create a phase portrait for the system.



16) Create the phase portrait for the nonlinear system given

$$\begin{aligned} \dot{x} &= xy \\ \dot{y} &= x+1 \end{aligned}$$



17) If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ gives complex eigenvalues, what must be true about the product $b \times c$?

Project

18) Describe in words what you did for your project.