GGAM Prelim Exam 1-00

ODE-119A

Problems + Solutions

#6 (a) Prove that

$$E = \frac{1}{2}\dot{x}^2 + P(x)$$

is constant along solutions of

$$\ddot{x} = -P'(x)$$

(b) Consider the 2nd order ODE

$$\ddot{x} = -\cos x + 5x^2. \qquad (*)$$

~~Find a~~ Use the energy to find a first order ODE that describes solutions of (*). Demonstrate that solutions of this scalar ODE are indeed solutions of (*). ~~Discuss the correspondence of the initial conditions~~

(c) Discuss the correspondence of the initial conditions

(#7) Assume $\underset{\sim}{x}(t) = (x, y)$ satisfies the DE

$$\frac{dx}{dt} = \lambda x + y + y^2 \qquad (1)$$

$$\frac{dx^2}{dt} = \lambda x,$$

where $\lambda > 0$ is an arbitrary constant.

(a) Find the matrix A such that the linearization of this system at $(0,0)$ is given by

$$\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}. \qquad (2)$$

(b) Find a formula for the solution set of (2)

(3) graph the solution set in the xy-plane (That is, graph the phase portrait.)

#8 Prove that the point $(0,0)$ is an asymptotically stable rest point for system

$$\dot{x} = 4x^3 + 2xy^{12}$$

$$\dot{y} = 12x^2 y^{11}$$

(That is, prove that all solutions tend to $(0,0)$ as $t \to +\infty$)