Case $n=1$

$P_n = \mathbb{C}[y_i]$, $i=1..2n$

$W_n = \mathbb{C}\langle y_i \rangle$

$\underbrace{[y_i, y_j]}_{2} = \alpha_{ij} \cdot 1$

$\mathrm{Aut}\, \mathbb{C}[y_1, y_2]$

$\varphi : \mathbb{C}[y_1, y_2] \circlearrowleft$

$\mathrm{Jac}(\varphi) = 1$

Th. (Shafarevich)

$y_1 \to y_1$
$y_2 \to y_2 + a y_1 + b \hookrightarrow$

$y_1 \mapsto y_1$
$y_2 \quad\; y_2 + P(y_1)$
$\forall P \in \mathbb{C}[y]$

$\downarrow$

$y_i \to \sum a_{ij} y_j + b_i$

$\det(a_{ij}) = 1$

$SL(2,\mathbb{C}) \ltimes \mathbb{C}^2$

free product

Th. (J. Dixmier, L. Makar-Limanov)

Same for W_1

R - commut. ring

$\varphi : W_n \otimes R \to W_n \otimes R$
autom. of alg. $/R$

$R / \mathbb{Z}/p\mathbb{Z}$ $\qquad p \cdot 1 = 0$

Center $(W_n \otimes R)$
$= R[\hat{y}_1^p, \ldots, \hat{y}_{2n}^p]$

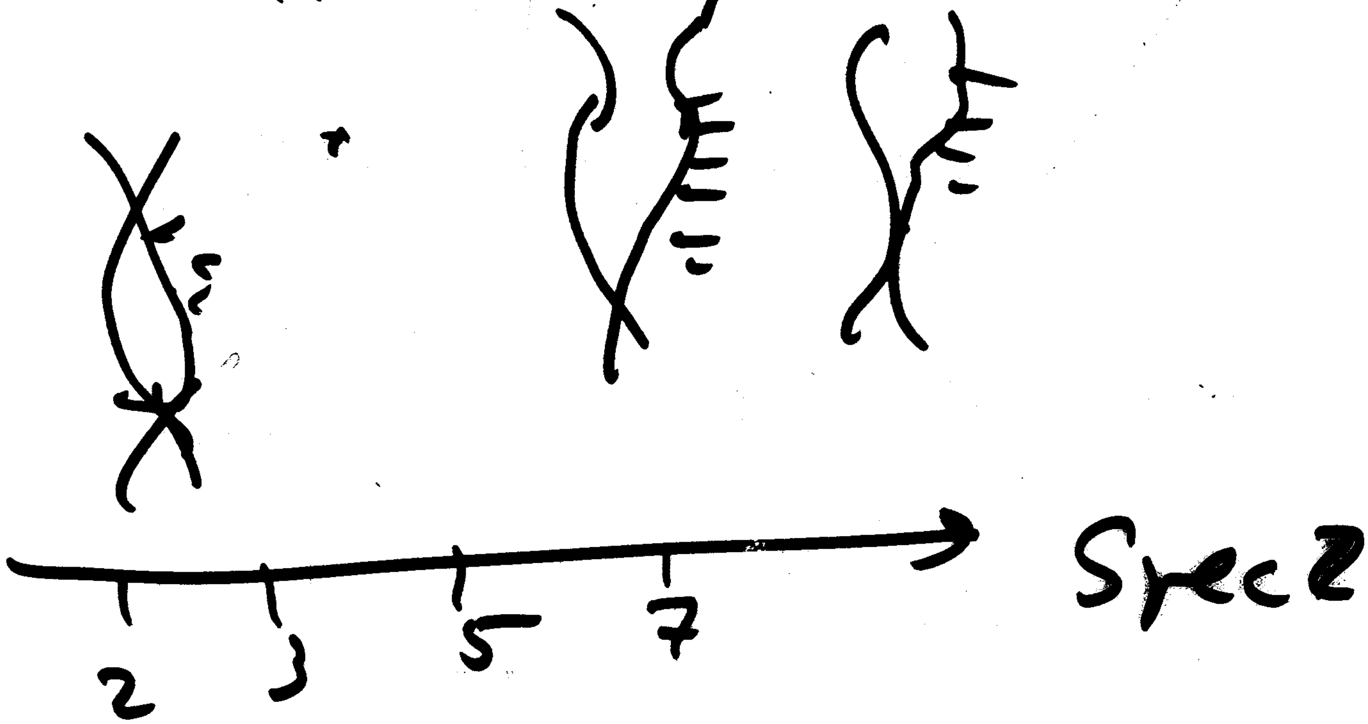
Ex. $\quad x^p \in \mathrm{Center}\ W_n \otimes \mathbb{Z}/p$

$[x, x^p] = 0$

$[\frac{\partial}{\partial x}, x^p] = p x^{p-1} = 0$

$(\underline{\mathrm{Aut}}\, W_n)^{\in M}$ $(\underline{\mathrm{Aut}}\, P_n)^{\in M}$

/ finitely gener. rings



/ nilpotents / few primes

Constructible iso.

$\mathrm{Der}(W_n \otimes \mathbb{Q}) \cong \mathrm{Der}(P_n \otimes \mathbb{Q})$

deg $\delta\varphi(\hat{y}_i) = n$ <

$\delta\varphi(\hat{y}_i^p) =$

$\delta\varphi(\hat{y}_i)\,\varphi(\hat{y}_i)^{p-1} + \varphi(\hat{y}_i)\,\delta\varphi(\hat{y}_i)\,\varphi(\hat{y}_i)^{p-2}$

$+ \cdots \quad \varphi(\hat{y}_i)^{p-1}\,\delta\varphi(\hat{y}_i)$

$a = \varphi(\hat{y}_i)$

$b = \delta\varphi(\hat{y}_i)$

Observation

$\hat{y}_i$ locally ad-nilpot

$(\mathrm{ad}\,\hat{y}_i)^{\gg 1}$ ($\forall$ element of W) $= 0$

$\big(\mathrm{ad}\ \underbrace{\varphi(\hat{y}_i)}_{a}\big)^{\gg 1}\big(\underbrace{\delta\varphi(\hat{y}_i)}_{b}\big) = 0$

$(\mathrm{ad}\,a)^M(b) = 0$

$a^M b + (-1)\binom{M}{1} a^{M-1} b a + \cdots$

$\pm b a^M = 0$

want

$a^p b + a^{p-1} b a + \cdots + b a^p = 0$
mod p

$p > M$

$$\left(\frac{\partial}{\partial x} + f(x)\right)^p$$

$$= \left(\frac{\partial}{\partial x}\right)^p + (f(x))^p \mod p$$

$$p > \deg f$$

$$D \in R \otimes W_n$$

$$(\operatorname{ad} D)^M (\hat{y}_i) = 0 \quad i = 1, 2$$

$$\longrightarrow D^p \in \text{Center}\,(R \otimes W_n$$

$$\sqrt[p]{D^p} \subset R \otimes P_n$$

Aut $P_n \longrightarrow$ Aut W_n

$R / \mathbb{Z}/p\mathbb{Z}$

$$R \otimes W_n$$
$$\cup$$
$$\text{Center} = R[q_i^p]$$

bundle of $\cong \mathrm{Mat}(p^n)$ "matrix algebras"

Azumaya algebra

S scheme

Azum. alg $/S$

A/S vector bundle rk ≥ 1

of assoc. algebras

$A \otimes A^{opp} \to End(A)$

is <u>iso</u>.

$Bun(S)$ vect. bun rk ≥ 1

$\begin{matrix} \mathcal{E} \\ \downarrow \\ S \end{matrix} \mapsto$ $End(\mathcal{E}) = \mathcal{E}^* \otimes \mathcal{E}$ Az. alg.

$Br(S) = $ Azu alg $/ \sim$. Morita equiv.

$S / \mathbb{Z}/p\mathbb{Z}$

Gabber:

$$\Omega^1_{abs} / d\Omega^0_{abs} \longrightarrow Br(S)$$

$\sum f_i \, dg_i \; /$ $(f_1+f_2)\,dg = f_1\,dg + f_2\,dg$

$-\text{''}-$ for g

$f\,d(g_1 g_2) = f g_1\,dg_2 + f g_2\,dg_1$

$f\,dg$ $\quad$ $\cancel{f}\,d\,1 = 0$

(f, g)

$x^p = f$ $\quad$ $[y, x] = 1$

$y^p = g$

$x^i y^j \quad 0 \le i, j \le p-1$

Better conjecture

$\mathbb{A}^{2n} \supset L$ alg. Lagrangian smooth

$\pi_1(L) = 0$

$(L \simeq \mathbb{A}^n)$

$\exists$ canonical module

M_L / W_n

Support $(M_L \otimes \mathbb{Z}/p\mathbb{Z})$ / center

$=$ Frobenius $(L \otimes \mathbb{Z}/p\mathbb{Z})$

minimal possible rank

Multiplicative version

$(\mathbb{C}^\times)^2$

$$\{x, y\} = xy$$

$$\omega = \frac{dx}{x} \wedge \frac{dy}{y}$$

$$M_{\lambda,\mu}(x,y) = (\lambda x, \mu y)$$

$$\lambda, \mu \in \mathbb{C}^{\times}$$

$$M_{\lambda_1,\mu_1} \cdot M_{\lambda_2,\mu_2} = M_{\lambda_2,\mu_2} \cdot M$$

$$S \cdot M_{\lambda,\mu} = M_{\cancel{\lambda\mu}^{-1},\mu} \cdot S$$

$$T \cdot M_{\lambda,\mu} = M_{\lambda\mu,\mu} \cdot T$$

$$\mathcal{U} \cdot M_{\lambda,1} = M_{\lambda,1} \cdot \mathcal{U}$$

$$M_{1,\mu} \cdot \mathcal{U} \cdot M_{1,\mu^{-1}} \cdot \mathcal{U} =$$

$$= \mathcal{U} \cdot M_{1,\mu} \cdot \mathcal{U} \cdot M_{1,\mu^{-1}}$$

Birat. autom.

$S(x,y) = (qy^{-1}, x)$

$T(x,y) = (xy, y)$

$\mathcal{U}(x,y) = (x\cdot(1+y)^{-1}, y)$

---

relations

$S^4 = 1$

$S^2 = (ST)^3$ | $SL(2,\mathbb{Z})$

$\mathcal{U}T = T\mathcal{U}$

$T\cdot\mathcal{U}S^2\cdot\mathcal{U}S^2 = 1$

$(S^{-1}\mathcal{U})^5 = \underline{1}$

Conj. Aut Birat.

$(\mathbb{C}^\times)^{2n}$ y_i

Preserving $\{y_i, y_j\} = a_{ij} y_i y_j$

$\downarrow$ maps canon

Aut "Birat"

$\hat{y}_i \hat{y}_j = \hat{y}_j \hat{y}_i \, q^{a_{ij}}$

$Sp(2n, \mathbb{Z})$

$\mathcal{U}$ on first 2 coord.

generate Γ_n

$\Gamma_n \longrightarrow \mathrm{Birat}\,(\mathbb{C}^\times)^{2n}$

Birat aut. of q-deform.

$\hat{y}_i \hat{y}_j = q^{\alpha_{ij}}\, \hat{y}_j \hat{y}_i$

$q = \sqrt[N]{1}$

Center $= \mathbb{C}[\hat{y}_i^{\,N}]$

Bundle of matrix alg. $/ \ (\mathbb{C}^\times)^{2N}$

$\forall \gamma \in \Gamma_n$

$\forall N$

aut. of Matrix $N \times$
als

$$\downarrow$$

$$(\mathbb{C}^\times)^n \circlearrowleft \gamma$$

Fock–Goncharov:

$\pi_1(\Sigma$ – punctu$) \to GL(k, \mathbb{C})$ /adj

surface

$\iint$ 2...

Birat

Coords $\underline{(\mathbb{C}^\times)}$

~~Teich.~~ Teich. $\mathrm{Aut}(\Sigma) \to$ Birat – Γ_{lang}

$\underline{\mathrm{Aut}}\, W_n$, $\underline{\mathrm{Aut}}\, P_n$

group ind-affine schemes $/\mathbb{Z}$

$$\varphi(\hat{y}_i) = \sum \varphi_i^{j_1 \ldots j_{2n}} \hat{y}_1^{j_1} \ldots \hat{y}_{2n}^{j_{2n}}$$

$$\varphi^{-1}(\hat{y}_i) = \sum (\tilde{\varphi})_i^{j_1 \ldots j_{2n}} \hat{y}_1^{j_1} \ldots \hat{y}_{2n}^{j_{2n}}$$

$\varphi_{\cdot}^{\cdot\cdot}$, $\tilde{\varphi}_{\cdot}^{\cdot\cdot}$ variables

$\varphi_i^{j_1 \ldots j_n} = 0 \quad \sum j_\alpha \geq M$

$\tilde{\varphi}$

$$\varphi \circ \varphi^{-1} = \varphi^{-1} \circ \varphi = \mathrm{id}$$

φ, $\tilde{\varphi}$ preserve ω, τ.

Preserves Poisson str.

$\mathbb{Z}/p^2\mathbb{Z}$

A_t Center(A_0).

Poisson $\{\,,\,\} = \lim_{t \to 0} \frac{(f,g)}{t}$

$$\left[\left(\frac{\partial}{\partial x}\right)^p, x^p\right] = \sum_1^p k! \binom{p}{k}^2$$

$x^k \left(\frac{\partial}{\partial x}\right)^k$

$= p! \mod p^2 = (-p) \mod p^2$

$\left(-\frac{1}{p}\right)[\,,\,] = 1 \mod p$

$\hat{y}_1, \hat{y}_2 \longrightarrow \lambda \hat{y}_1, \lambda^{-1} \hat{y}_2$

$\hat{y}_1^p, \hat{y}_2^p \longrightarrow \lambda^p \hat{y}_1, \lambda^{-p} \hat{y}_2$

Extract p.th root?

$R / \mathbb{Z}/p\mathbb{Z}$

$Fr : R \longrightarrow R$

$a \longrightarrow a^p$

$(a+b)^p = a^p + b^p \bmod p$

<u>Claim</u> Coefficients for Aut of P_n / R belong to $\underline{R^p}$.

if $R/\mathbb{Z}/p$ is algebra

of fun. on smooth alg. variety

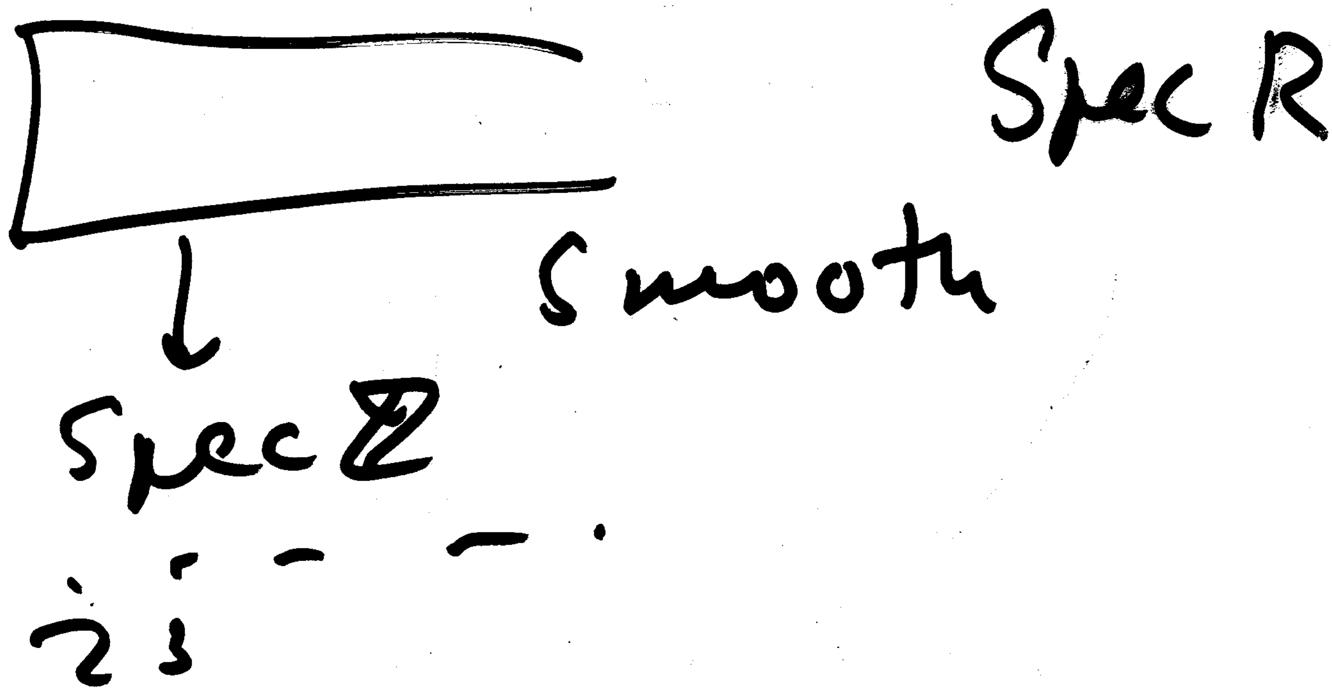
$\mathbb{Z}/p\mathbb{Z}[x_1, \ldots, x_n]$

$f \in R$ is pth power $f \in R^p$

$\Leftrightarrow \delta f = 0$

$\forall \delta \in \mathrm{Der}\, R$

$\frac{\partial}{\partial x}(x^p) = 0$



$\mathrm{Der}(R/\mathbb{Z}_p) = (\mathrm{Der}\, R) \otimes \mathbb{Z}/p$

$\delta \in \mathrm{Der}\, R$

φ universal auton

$\mathrm{Aut}(R \otimes K_n)$

<u>Lemma</u>: for $p \gg 1$.

$$\delta\varphi(y_i^p) = 0 \mod p$$

Conjecture

on smooth pieces
extract p-th root
$\leadsto$ Aut of P_n mod p
$\forall p$, depend on parameter

This is reduction
of Aut of char $=0$.

True for affine transform
$Sp(2n, \mathbb{R}) \ltimes \mathbb{R}^{2n}$
polyn.

$x_i \longrightarrow x_i$

$$\frac{\partial}{\partial x_i} \longrightarrow \frac{\partial}{\partial x_i} + \partial_i F(x)$$

$\deg F < p$

$\varphi^{*} \in \operatorname{Aut} P_n \otimes R$

aff. space $\mathbb{A}^{2n}_R \circlearrowleft$

$\varphi^{*}(\sum p_i\, dq_i)$

$= \sum p_i\, dq_i + dF$

$\otimes A_{p_i, q_i}$ $/ R\otimes \mathbb{Z}/p$ = Weyl algebra

$\uparrow q_i^p$

$\longrightarrow$ φ mod p gives Morita Autoequivalence.

L = graph of symplecto

$\longrightarrow$ my conj about Aut.

How to get rid of primes.

Def: $L \subset \mathbb{C}^{2n} = T^*\mathbb{C}^n_{x_i}$ is <u>tame</u>

if $\exists$ gener. function

$f : \underset{x}{\mathbb{C}^n} \times \underset{\xi}{\mathbb{C}^M} \longrightarrow \mathbb{C}$ polynom

$$L = \left\{ \left(x, \frac{\partial f}{\partial x}\right) \;\middle|\; \frac{\partial f}{\partial \xi} = 0 \right\}$$

$$M_L = \int e^{f(x,\xi)}\, d\xi$$

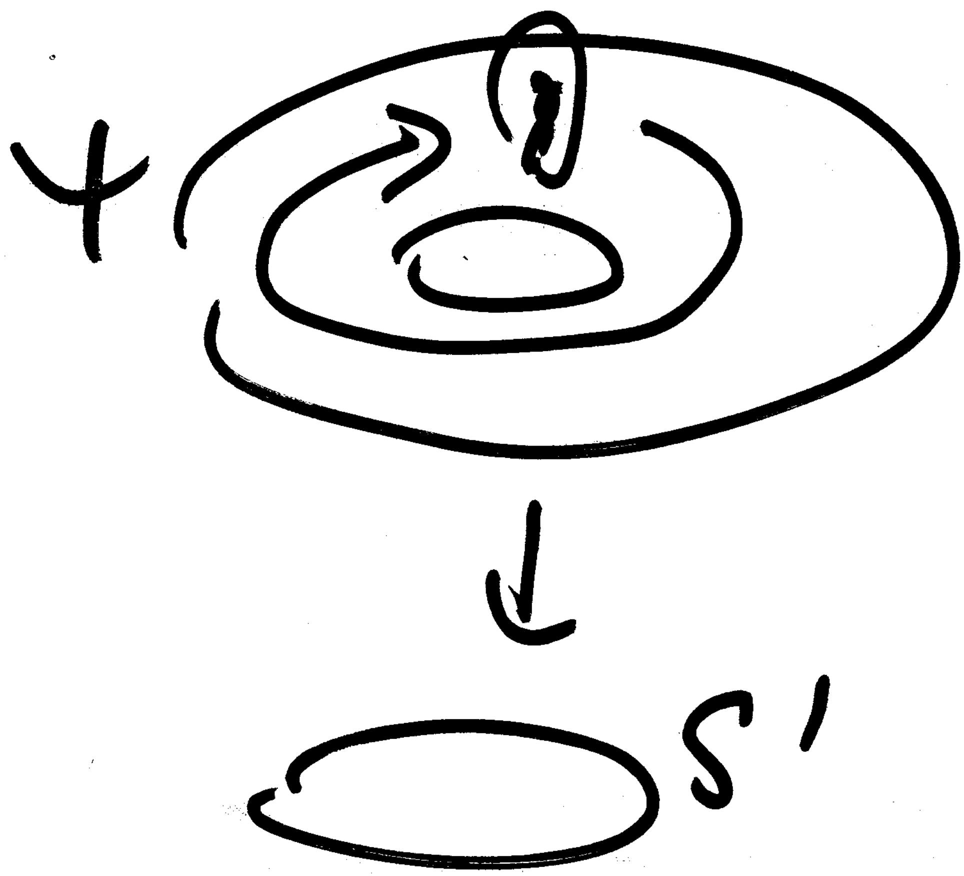
$$\{x, y\} = xy$$

$$\hat{y}\hat{x} = q\,\hat{x}\hat{y} \qquad q \in \mathbb{C}^{\times}$$

$\psi^*: \Sigma \circlearrowleft$

fixed points under ψ^*

$= \pi_1(3\text{ dim}) \longrightarrow GL$



conjecture $\forall N$

$\sum$ Tr (aut of matrix algebras)

fixed points

is CS invariant

$\stackrel{?}{=}$ Tr "H^*(. . .)"

in

cohomology of quantum algebra

= lesser form

$q = \sqrt[N]{1}$ prim. root

$$\prod_{j=1}^{N-1} \left(1 - x^{\frac{1}{N}} q^{j}\right)^{j} \cdot \prod_{j=1}^{N-1} \left(1 - (1-x)^{\frac{1}{N}} q^{j}\right)^{j} = Z$$

$Z \in \mathbb{Q}\left(q, x^{\frac{1}{N}}, (1-x)^{\frac{1}{N}}\right)$

Claim $Z^{\frac{1}{N}} \in$ — " —

$N = 1 \ (2)$

joint work with A. Belov-Kanel. On Algebraic (de)quantization and finite characteristic

Additive part $n \geq 1$ $n \in \mathbb{Z}$.

W_n $\mathbb{Z}\langle x_1, \dots x_n, \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \rangle \Big/ \ [x_i, x_j] = 0, \ [\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j}] = 0, \ [\frac{\partial}{\partial x_i}, x_j] = \delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

P_n $\mathbb{Z}[q_1, \dots, q_n; p_1 \dots p_n]$ product + Poisson bracket $\{f, g\} = \sum_{i=1}^n \left(\frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} - \frac{\partial g}{\partial p_i} \frac{\partial f}{\partial q_i} \right)$

$\{f, gh\} = \{f, g\}h + \{f, h\}g$

$\{f\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0$.

$P_n = \mathbb{Z}[y_i]$ $i = 1 \dots 2n$ $\ \{y_i, y_j\} = \omega_{ij}$ $\ \omega_{ij} = (\ )$ - ω_{ij} sympl. form

$W_n = \mathbb{Z}\langle \hat{y}_i \rangle / [\hat{y}_i, \hat{y}_j] = \omega_{ij}$

Usual relation. W_n filtered $W_n^{\leq N}$ total degree $\bigoplus \mathrm{Gr}_N W_n^{\leq N} = P_n$ co.

homog. elements $[f, g] \in m_1 + m_2 - 2$ - filtr. term, leading term is bracket.

More direct relation.

Conjecture: $\forall$ field k char $k = 0$ (..$k = \mathbb{C}$)

$\mathrm{Aut}(W_n \otimes k / k) \underset{\text{canon.}}{\simeq} \mathrm{Aut}(P_n \otimes k / k)$

algebras over k.

$\mathrm{Aut}(\mathrm{Aut}(P_n \otimes k)/k)) \supset k^\times$ $\ \hat{y}_i \to \lambda \hat{y}_i$ $\ i = 1 \dots n$ $\ \underbrace{\ \ \ }_{\text{2 resum}}$ $\S .3.$

evidences. $\longleftarrow$ different 2 reasons

$\longrightarrow$ 1 reason.

$\{\ ,\ \} = \alpha^{-1} dy_1 \wedge dy_2$ 2 form symplect.

Case $n = 1$: $\mathrm{Aut}\, \mathbb{C}[y_1, y_2]$ ~~preserves~~ Jacobian = 1.

Th. (Shafarevich). $\mathrm{Aut}_{\mathrm{Jac}=1} \mathbb{K}[y_1, y_2] = $ $\begin{matrix} y_1 \to y_1 \\ y_2 \to y_2 + ay_1 + b \end{matrix}$ $\hookrightarrow$ upper-triangular transf $\begin{matrix} y_1 \\ y_2 \end{matrix} \longrightarrow \begin{matrix} y_1 \\ y_2 + f(y_1) \end{matrix}$ $\ \forall f \in k[y_1]$ polynomial

is free product over subgroup.

affine δ trans. $\begin{matrix} y_1 \\ y_2 \end{matrix} \longmapsto \sum a_{ij} y_j + b_i$ $\ \underbrace{2 \times 2 \ SL(2) \ SL(2, k) \ltimes k^2}$

Th. (Dixmier, L. Makar-Limanov) $\mathrm{Aut}\, W_1 \otimes k$ — same. $\ \begin{matrix} \hat{y}_1 \to \hat{y}_1 \\ \hat{y}_2 \to \hat{y}_2 + f(\hat{y}_1). \end{matrix}$

$\underline{\mathrm{Aut}}\, W_n$ ind. affine group schemes $/\mathbb{Z}$ $\quad \varphi : \hat{y}_i \to \sum \varphi_i^{j_1 \dots j_{2n}} \cdot \hat{y}_1^{j_1} \dots \hat{y}_{2n}^{j_{2n}}$ $\quad \varphi_i^{j_1 \dots j_n}$ $\ (\varphi^{-1})_i^{j_1 \dots j_n}$ con vari

$\underline{\mathrm{Aut}}\, P_n$ $\quad \varphi^{-1} = \ \to \sum (\varphi^{-1})_i^{j_1 \dots j_n}$

vanish. $\sum |j_i| \geq N$

finite # of variables

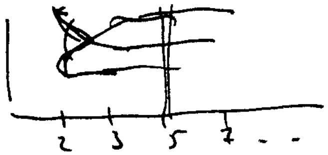
Equation $\varphi^{-1} \circ \varphi = \varphi \circ \varphi^{-1} = id$ | ring $R_{W_n}^{\leq N}$

Same for P_n. $\quad R_{P_n}^{\leq N}$

$R_{W_n}^{\leq N}$ finitely gen. rings.

$R_{P_n}^{\leq N}$

let. $\otimes \mathbb{Q}$ More precisely.



$R^{\varepsilon N}_{W_n} \otimes \mathbb{Q}$ / nilpotents $\overset{?}{\cong}$ $R^{\varepsilon N}_{P_n} \otimes \mathbb{Q}$ / nilpotents

$\to \mathrm{Spec}\, \mathbb{Z}$

| nilpotents $\neq$ to each other. Lie algebra $\quad \mathrm{Der}(W_n \otimes \mathbb{Q}) \underset{\text{iso}}{\not\cong} \mathrm{Der}(P_n \otimes \mathbb{Q})$

equisingular for large primes p.

universal automorphism

R in char p. $\quad R$ is fin. generated $\mathbb{Z}$-in

$(\mathrm{Aut}\, W_n)^{\varepsilon N} \otimes_{\mathbb{Z}} \mathbb{Z}/p\mathbb{Z}$

well-known observ.

$\varphi: \cancel{\mathrm{Aut}}\ W_n \otimes R \to W_n \otimes R$

Center $W_n \otimes R = R[\hat{y}_1^p, \ldots, \hat{y}_{2n}^p]$

basic calculat. $\quad x^p \in_{\text{cent}} \mathbb{Z}/p\,\{x, \frac{\partial}{\partial x}\} / (\frac{\partial}{\partial x}, x) = 1 \quad$ not p-th powers not central.

$[x^p, x] = 0$

$[x^p, \frac{\partial}{\partial x}] = -p x^{p-1} = 0 \mod p.$

get autom. of the center! it preserves Poisson bracket mod p!

Claim $p \gg 1$ give n:

$\hat{x}_i^p \mod p^2$

$\mathbb{Z}/p^2\mathbb{Z}$

$[(\frac{\partial}{\partial x})^p, x^p]_{\mod p^2} = \sum_{k=1}^{p} \frac{1}{k!} \binom{p}{k}^2 x^k (\frac{\partial}{\partial x})^k = \frac{?}{p!} p! \mod p^2$

$= (-p) \mod p^2$

alg. der. on part $(-\frac{1}{p})[(\frac{\partial}{\partial x})^p, x^p] = 1 \mod p.$

A_t

Center A_0 $\{\ ,\ \}$ $\frac{1}{t}[f_t, g_t] \mod t$.

Poisson bracket

Naive idea: get $\mathrm{Aut}(P_n \otimes R)/p \quad \times p \quad$ appear from same

$\hat{y}_1, \hat{y}_2 \to \lambda \hat{y}_1, \lambda^{-1} \hat{y}_2$

$\hat{y}_i^p \to \lambda^p \hat{y}_i^p, \lambda^{-p} \hat{y}_i$

should

p-th power

Extract p-th roots?

$\forall$ ring of char p. $\quad R$

$\lambda \mapsto \lambda^p$ is an endomor

$(\lambda + \mu)^p = \lambda^p + \mu^p \mod p.$

Cover $\mathrm{Spec}\, R^{\varepsilon N}$ by finite # of smooth pieces $P_n / \mathrm{Spec}\, \mathbb{Z}$

(mod nilpotents)

On smooth alg in char p.

$\mathbb{Z}/p[x_1 \ldots x_n]$

$\mathrm{Der}.(x_i^p) = 0.$

$f \in R^p \iff D(f) = 0$
$\forall$ deriv. D of

φ autom. dep. on parameters δ der. of M

Lemma $\delta\varphi(\hat{y}_i^{\,p}) = 0$ mod p for $p \gg 1$.

Leibniz. $\delta\varphi(\hat{y}_i^{\,p}) = \sum_{j=0}^{p-1} \varphi(\hat{y})^j \, \delta\varphi(\hat{y}_i) \, \varphi(\hat{y}_i)^{p-j-1}$

$\deg \delta\varphi(\hat{y}_i) \le N$.

Observation: $\hat{y}_j$ locally ad-nilpotent elem of W_n

$(\operatorname{ad} \hat{y}_i)^{\text{large}} (\forall$ elem of $W_n) = 0$.

$(\operatorname{ad} \varphi(\hat{y}_i))^M (\delta\varphi(\hat{y}_i)) = 0$
also ad-nil.

$a = \varphi(\hat{y}_i)$
$b = \delta\varphi(\hat{y}_i)$

mult left on $\prod a^i \to a^{p-1-M-i}$

$\sum$(appropriate weights binom coeffs) $= \ldots$

$(\operatorname{ad} a)^M (b) = 0$

$a^M b - \binom{M}{1} a^{M-1} b a + \ldots \pm b a^M = 0$

want to prove
$a^{p-1} b + a^{p-2} b a + \ldots + b a^{p-1} = 0$

Conjecture: on smooth pieces. extract p-th root:
aut mod p $p \gg 1$ of p-th aut. in der $= 0$!

This is reduction mod p of

True for affine. transformation. explicit. (same autom)

$\frac{\partial}{\partial x_i} \to \frac{\partial}{\partial x_i} + \frac{\partial F}{\partial x_i}$

$x_i \longrightarrow x_i$

$F \in \mathbb{Z}[x_1 \ldots x_n]$
$p > \deg F$.

$\left(\frac{\partial}{\partial x} + f(x)\right)^p = \left(\frac{\partial}{\partial x}\right)^p + (f(x))^p$ mod p $p > \deg f$.

No idea why.

~~Inverse~~ ~~construct.~~

More generally: $D \in R \otimes W_n$ $R / \mathbb{Z}$

Diff. operator ad nilpotent. $(\operatorname{ad} D)^M (\hat{y}_i) = 0$ $\forall i = 1 \ldots 2n$.

$\longrightarrow D^p \in \text{Center}_{\text{mod } p}$ $p > M$

$\sqrt[p]{\text{coeff.}}\ D^p$:

elem th Poisson algebra
Canonical assoc to D "Symbol"

Φ def over some $\mathbb{Z}$ ... $\sqrt[p]{\ldots}$ deg p.

$\underline{\mathrm{Aut}}\ P_n \to \underline{\mathrm{Aut}\, W_n}$: 1) / parameter space

mod p W_n is Azumaya algebra / Center $W_n = k[\xi_i^p, \eta_i^p]$.

(bundle of matrix algebras $p^n \times p^n$)

scheme: $A_3(S)$ A/S vector bundle $r > r-4 \geq 1$

product $A \otimes A^{opp} \to \mathrm{End}(A)$ is iso.

monoid $A \otimes B$

$\mathrm{Bun}_+(S)$ vector bundle of $rk \geq 1$. $A = \mathrm{End}(V)$

$\hookrightarrow A_3(S)$ $A \otimes A^{opp}$ trivial.

$\sim$ iso. class $A \otimes B = \mathrm{End}(S)$.

$Br(S)$ Brauer group

$S/\mathbb{Z}/p\mathbb{Z}$ $\Omega^1_{abs,\, S}/d\Omega^0_{abs} \longrightarrow Br$

1-forms $f\,dg$ $(f,g) \longrightarrow$ $A(f,g)$ algebra $x^p = f$, $y^p = g$, $(x,y) = 1$. Azumaya. basis $x^i y^j$ $0 \leq i,j \leq p-1$

$cl(A_{(f,g)}) = -cl(A_{(g,f)})$ opposite alg.

$A_{(0,f)} = 0$ $x \Rightarrow \begin{pmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ 0 & & & \end{pmatrix}$, $y \to \begin{pmatrix} f^0 & & \\ & f^1 & \\ & & \ddots \end{pmatrix}$

$A_{f_1 f_2, f_3} \otimes A_{f_2 f_3, f_1} \otimes A_{f_3 f_1, f_2}$ = matrix algebra. $\simeq A_{0,f_1} \otimes A_{0,f_2} \otimes A_{0,f_3}$

$x_1^p = f_2 f_3$ $x_2^p = f_2 f_3$ $(x_1 - y_2 y_3)^p = 0$ $(x_2 - y_1 y_3)^p = 0$

$y_1^p = f_3$ $y_3^p = f_1$ ... $y_1^p = f_1$ $y_2^p = f_2$ ...

$\varphi : \mathbb{A}^{2n} \circlearrowleft$ symplectomorphism.

$\varphi^* \left(\sum p_i dq_i\right) = \sum p_i dq_i + df$ Morita equivalence.

elementary steps.. $\setminus$ finite # of primes in denominator.!

Problem: we get Morita Autoequivalence. $W_n \otimes R/_{\text{mult } pp}$ $\cong$ $W_n \otimes R/_{..}$

some autom. ?

Deform. quantization. $\forall$ rat. can $\to$ can quantized $/\mathbb{Q}[[\hbar]]$ $\mathrm{Aut}(\mathbb{A}^n)$ ... if true.. $\longleftrightarrow$ ... bounded degree? invariant under dilation. ok.. for den

Better conjecture:

$\mathbb{C}^{2n}$ $\mathbb{A}^{2n} \supset L$ smooth Lagrangian algebraic submanifold in numbers.

sym. vector space

$\pi_1(L) = $ trivial. $(L \simeq \mathbb{C}^n)$

$\exists$ canonical module M_L over Weyl algebra W_n

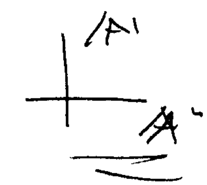
Support $M_L \otimes \mathbb{Z}/p\mathbb{Z}$ / center $=$ Frobenius (L)

rank: minimal possible.

My conj. $\Leftarrow$ graphs of symplectomorphism.

How to get rid of p-s? $L \subset T^*\mathbb{A}^n$ generating functions!

L tame: if $\exists$ $f: \mathbb{A}^n \times \mathbb{A}^M \to \mathbb{C}$
(x, ξ)

in terms on the choice of 

$\frac{1}{2}$ of $2n$ coord in x.

$= \{(x, \partial_x f) \mid \partial_\xi f = 0\}$.

$\mapsto$ module $M = \int e^{f(x,\xi)} d\xi^M$

All examples known to me are tame.

Multiplicative version. $\{x, y\} = xy$ $\omega = \frac{dx}{x} \wedge \frac{dy}{y}$

2 variables: $\mathbb{C}^{x} \times \mathbb{C}^{y}$

Birational automorphism

Conj. — - - 1 —

e.g. Birat. aut.

$A = \mathbb{Q}\langle \cancel{x_1, \ldots} y_1, \ldots y_{2n} \rangle$

preserving $\{y_i, y_j\} = \omega_{ij}\, y_i y_j$

$\downarrow$ map

A_q / $\hat{y}_i \hat{y}_j = q^{\omega_{ij}} \hat{y}_j \hat{y}_i$

"Birat. autom."

primitive

$q = \sqrt[N]{1}$

Center $A_q = \mathbb{C}[\hat{y}_i^{N}]$ / $(\mathbb{C}^{\times})^{2n}$

Bundle of matrix algebras

auto. of the center is "the same"..

special type

$\varphi : (\mathbb{C}^{\times})^{2n} \circlearrowleft$ birat. auto.

matrix algebra $N^n \times N^n$.

a set. auto. of

at $\forall$ fixed point auto of m

Special part. S, T, U : group

Cluster algebra

Application to Chern-Simons? :

Σ surf. - π_1

rep. $\pi_1(\Sigma) \to GL(m, \mathbb{C})$, Goncharov Fock, $\mathbb{Z}$

m. str. (birational) $(\mathbb{C}^{\times})^{2}$

coordinate

Teich. ST, U,

$$\sum_{p \text{ fixed points of } \varphi} \mathrm{Tr}\left(\tilde{\varphi}^{(N)}(p)\right)$$

depends on N.

$q = e^{\frac{2\pi i}{N}}$

$N \to \infty$ $\exp\left(\sum \mathrm{Li}(\cdot \cdot)\right)$

Aut of matrix alg.

$\rightarrow$ aut. of vector sp.

Super-Trace in "Cohomology"

analogue

ii: $\prod_{i=1}^{N-1} (1 - x^{\frac{1}{N}} q^{i})^{j}$. $\prod (1 - (1-x)^{\frac{1}{N}} q^{j})^{j} = z \in \mathbb{Q}(q, x^{\frac{1}{N}}, (1-x)^{\frac{1}{N}})$

$q \to$ ... $q^{\frac{1}{N}}$: $z^{\frac{1}{N}} \in$ same fields.

$S(x,y) = (q y^{-1}, x)$

$yx = q xy$

$T(x,y) = (xy, y)$

$\mathcal{U}(x,y) = (x \cdot (1+y)^{-1}, y)$

Соотношение:

$S^4 = 1$

$S^2 = (ST)^3$

$\mathcal{U}T = T\mathcal{U}$

$T \cdot \mathcal{U}S^2 \cdot \mathcal{U}S^2 = 1$

$(S^{-1}\mathcal{U})^5 = 1$

Дополнительные образующие

$M_{\lambda,\mu}(x,y) = (\lambda x, \mu y)$ $\lambda, \mu \in \mathbb{C}^\times$

Соотношение:

$M_{\lambda_1,\mu_1} \cdot M_{\lambda_2,\mu_2} = M_{\lambda_1\lambda_2, \mu_1\mu_2}$

$S \cdot M_{\lambda,\mu} = M_{\mu^{-1},\lambda} \cdot S$

$T \cdot M_{\lambda,\mu} = M_{\lambda\mu, \mu} \cdot T$

$\mathcal{U} \cdot M_{\lambda,1} = M_{\lambda,1} \cdot \mathcal{U}$

$M_{1,\mu} \cdot \mathcal{U} \cdot M_{1,\mu^{-1}} \cdot \mathcal{U} = \mathcal{U} \cdot M_{1,\mu} \cdot \mathcal{U} \cdot M_{1,\mu^{-1}}$