

# Research Focus Groups in Mathematics

## Excerpt from the UC Davis VIGRE proposal

A Research Focus Group (RFG) consists of faculty, postdocs, graduate students and, mainly in the summer, undergraduate students, who collaborate in year-long research projects supported by a seminar series, mini-courses, and discussion groups. The topics will be selected from proposals solicited from the faculty of the Department of Mathematics and the Graduate Group in Applied Mathematics. Topics are expected to vary from year to year, but successful Research Focus Groups may continue for several years. However, we think that it is important to make it possible to start up new focus groups as new interests arise, e.g., with the hiring of new faculty members in our growing department. Less successful or too narrowly focused groups will be discontinued. A special effort will be made to include topics that reflect exciting new mathematical developments. Preference will be given to topics that reflect the strengths of the faculty and that have a strongly interdisciplinary component. The Research Focus Groups will hold regular activities aimed at graduate students, postdocs and a broad group of faculty.

The focus of activities of each group will be a weekly seminar which will alternate between local speakers and visitors. Talks will vary among survey talks by senior faculty, research talks by visitors, expositions of classical and foundational papers by graduate students, postdocs reporting on important current or recent results, as well as discussion of open problems by experts in the area. The seminar series will supplement rather than replace the current research seminar series of the Department. The seminars will be modeled after the topical “working seminar” series organized by some faculty. In fact, we expect that several Research Focus Group proposals will be built on the informal seminars that members of the department are currently running. Each Research Focus Group seminar will be listed as a seminar course in the MAT290 series, but, unlike regular MAT290’s, the lead faculty will receive 1 course credit per year. This is to recognize that the lead faculty will take on many additional responsibilities in organizing the other activities of the Research Focus Group.

The seminars will be organized as research seminars which offer one unit of credit to students who attend and participate regularly and three units for more advanced students who give a talk or talks. First year students attending these seminars will be able to get an overview of a field in which they can pursue research. More advanced students will learn about results in the area, and its techniques and problems. Postdocs and faculty will be exposed to new developments, as well as get an opportunity to meet and interact with students and with each other.

Associated to each seminar series will be a reading seminar. The reading seminar will be mainly aimed at getting graduate students up to speed on current research in the focus group area. Every few weeks a new subject or paper will be chosen for these reading seminars. A typical scenario would see participants taking turns presenting materials in a paper, with periodic guest lectures filling in missing background. The topics would be chosen by the participants in the research seminar series, with an emphasis on material that is accessible to graduate students beginning their individual research, and of interest also to postdocs

and more senior researchers. One of the goals of the reading seminar will be to encourage active participation, rather than passive absorption, by graduate students in the research seminar. Another is to give students and postdocs extensive opportunities to present work, their own and others, in an informal setting.

In addition to the research and reading seminars, each of the focus groups will run an informal week-long research conference each year. This will involve a mixture of local and invited speakers. A leading researcher in the area will be invited to Davis to give a series of 3-5 talks, with at least one aimed at a general campus audience, including undergraduates. Several other researchers will be invited to give a one-hour talk, and additional presentations will be made by graduate students and postdocs who have relevant new results. Such an informal conference will provide a forum for the graduate students, as well as postdocs, to present their current research, to absorb the mathematical culture, and to get practice in presenting research in a conference environment. More advanced undergraduates in the Research Focus Groups will also be invited to participate.

The Research Focus Groups will help PhD students to settle on an area of research and an advisor, thus reducing the time required to complete their degree. By focusing the students on an area of research early on, we hope to alleviate this problem. The Research Focus Groups will give students an opportunity to interact with one particular group of faculty very intensively, making the choice of an advisor less intimidating. Other reasons that may delay the completion of the PhD are procrastination in taking the qualifying exam, slow progress in the research project, and the actual writing of the thesis. We do not have a magic recipe to address all of these problems. However, stepping up advising efforts and encouraging students to participate in activities with a group of fellow students, such as those in the Research Focus Group, will help them concentrate on their goals.

The focus groups will function year round, but the activities in the summer will differ from those during the fall, winter, and spring quarters.

Graduate students and postdoctoral fellows will typically join a Research Focus Group focus in the fall, while undergraduate students are more likely to enter in the summer with the Undergraduate Research Experience program.

Focus groups activities during the fall, winter, and spring quarters will typically include the following: the research seminar, the reading seminar, and one or more collaborative research projects involving graduate students, postdocs, and faculty. Focus groups will also play a role in the recruitment (e.g., by hosting prospective graduate students, or by targeted attendance of certain conferences) and outreach activities of the VIGRE project. For example, undergraduate as well as graduate students will be offered the opportunity to participate as teaching assistants in the COSMOS program (<http://cosmos.ucdavis.edu>).

In the summer, each focus group will guide a group of undergraduate participants in reading and research work. Proposed research projects for undergraduate students will be part of the initial proposal for establishing a Research Focus Group. Focus groups will recruit undergraduate students for summer participation throughout the year, via five avenues:

- 1) Through the instructors: the VIGRE committee will ask all instructors to make announcements in their classes and post an announcement on their course webpage. We will

also ask instructors to transmit the names of potentially interested students to the VIGRE committee, who will then contact those students.

- 2) Through postings on the undergraduate information bulletin boards.
- 3) Through the Department's undergraduate advisers, and bulletin boards.
- 4) Through postings on the undergraduate majors "mathtalk" newsletter.

5) Through collaboration with various special programs on campus that aim to increase participation in math and science of students from underrepresented groups. Examples include the McNair Scholars program, designed to encourage students from groups often underrepresented in graduate programs to pursue doctoral degrees (<http://mcnair.ucdavis.edu>), the UC LEADS program (Leadership Excellence through Advanced Degrees, <http://grad-studies.ucdavis.edu/recruit/ucleads.htm>), which is a two-year program designed to identify educationally or economically disadvantaged undergraduates in sciences, engineering or mathematics who show promise of succeeding in doctoral degree programs, and the MURPPS program (Mentorships for Undergraduate Research Participants in the Physical and Mathematical Sciences, <http://maxwell.ucdavis.edu/~murpps>). Every undergraduate student will be assigned a mentor from the focus group, who will either be a faculty member, a Visiting Research Assistant Professor, or a VIGRE postdoctoral fellow. The undergraduate students will be given office space during the summer months to encourage daily interaction with each other, their mentors, and especially with the graduate students in the focus group.

Near the end of the summer we will organize an Undergraduate Research Experience closing festival with presentation of the undergraduate research projects, followed by a picnic offered by the Math Department to include all participants.

Undergraduate students will be encouraged to continue to participate in the focus group during the academic year. We hope that they will come to feel like welcome members of the Math Department. We also expect that many of those entering their senior year will consider continuing their research through a senior thesis project. Undergraduates who have completed a project will be directed to conferences where they can present their research in the form of a poster or a short oral presentation (such as the UC Davis Undergraduate Research Conference, the annual regional SIAM meeting, or the Pi Mu Epsilon session at the AMS/MAA summer MathFest) and to submit a report on their work to an appropriate journal (local journals such as *Explorations, an Undergraduate Research Journal*, or the *McNair Scholars Journal of UC Davis*, national journals such as the *Undergraduate Mathematics Journal* of the MAA, or a refereed research publication if the results warrant it). Their mentors will provide them with guidance in these activities, and in the case of off-campus conferences, funds requested in this proposal will be used to cover the travel expenses of the students. We believe that this will be an excellent investment. It will give the participating students the additional experience of presenting their work to a new audience, it will expose them to a wider variety of research topics, and it will also serve indirectly to enhance the visibility of the research programs at UC Davis to the many graduate-school-bound undergraduates from other schools that participate at such conferences.

Below, we have included brief descriptions of four examples of possible Research Focus

Group. They by no means exhaust the possibilities. Other topics for which there exists a strong interest in the department include General Relativity and Nonlinear Waves, Partial Differential Equations and Fluid Dynamics, and Applications of Harmonic Analysis to Images and Signals.

1) *Geometry and Topology, including Computational Aspects.*

Faculty participants: Joel Hass, Greg Kuperberg, Jesus De Loera, Abigail Thompson, Bill Thurston.

This Research Focus Group will revolve around a series of themes involving computations in 3-dimensional geometry and topology. The goal of the Group is to bring together a broad collection of faculty, postdocs and students, with interests in 3-manifold theory, computational geometry, and hyperbolic and differential geometry, and to set up joint activities with colleagues in engineering and computer science. The group will create a lively research environment for those working on problems in geometry and topology, foster interactions between mathematicians, computer scientists and engineers who work on geometric problems, and give a broader vision of the wide applicability of mathematical ideas to the graduate students of the Department.

The participating faculty typically have a combined total of seven or eight doctoral students. It is anticipated that 15 graduate students will participate in the group's activities, with some from related fields such as Engineering, Computer Science and Physics, and some beginning students who have not yet chosen a research field. In addition to the faculty listed, many of the group's seminars will be attractive for Davis faculty with related interests, such as Carlip in Physics, Farouki in Mechanical Engineering, and Gusfield in Computer Science, as well as to other mathematicians such as Fuchs, Khovanov, Mulase, Nachtergaele, and Schwarz. De Loera, Thompson and Thurston have already worked with undergraduates on undergraduate research projects, and it is anticipated that all five of the faculty listed above will do so through this group.

The topics that the group intends to study in its initial year are focused on computational aspects: The complexity of geometric and topological problems; Normal surfaces and their use in algorithms in 3-manifolds; Algorithms for recognizing manifolds and for determining knottedness and the complexity of such algorithms; Expander graphs - their use in computer science, their geometric properties, and their connections to the eigenvalues of the Laplacian on graphs and manifolds. Geometries and computations in groups; Computing geometric structures on 3-dimensional manifolds; Computing optimal shapes and isoperimetric inequalities.

Some papers and sources for an associated reading course: Epstein, Cannon, Holt, Levy, Paterson, and Thurston, *Word processing in groups*; Bern and Eppstein, *Mesh Generation and Optimal Triangulation*; Bjorner, *Topological Methods in Combinatorics*; Edelsbrunner *An incremental algorithm for Betti numbers of simplicial complexes on the 3-sphere*; Brooks, *Spectral geometry and the Cheeger constant - Expanding graphs*; Friedman, *Computing Betti Numbers via Combinatorial Laplacians*; Gromov, *Hyperbolic groups*; Jaco and Rubinstein, *Zero-efficient triangulations of 3-manifolds, PL Minimal surfaces in 3-manifolds*; Lubotzky, Phillips and Sarnak, *Ramanujan graphs*; Matveev, *Algorithmic classification of*

*three-dimensional manifolds: problems and results*; Reingold, Vadhan and Wigderson *Entropy Waves, the Zig-Zag Graph Product, and New Constant-Degree Expanders and Extractors*, Rubinstein, *Polyhedral minimal surfaces, Heegaard splittings and decision problems for 3-dimensional manifolds*.

2) *Spectrum and Dynamics in Classical and Quantum Many-Body Problems*.

Faculty participants: Janko Gravner, Bruno Nachtergaele, Alexander Soshnikov, Craig Tracy.

The activities of this Research Focus Group will be centered around three research themes: pattern formation and moving interfaces in interacting particle systems, universal power laws in random matrices and correlation functions of processes in random environments, and the dynamics of magnetic structures modeled by quantum spin systems. These are three topics of active research which have in common a close relation with statistical mechanics and probability theory. Students in the group will be given the opportunity to acquire a firm foundation in these areas.

Additionally, a reading seminar will explore new ideas in non-equilibrium statistical mechanics developed in the past few years by Gallavotti, Ruelle, Lebowitz, Jaksic & Pillet, and others. Introductory papers by these authors will start things off. Missing background material can be filled in by local experts and visitors. One of the goals of the reading seminar will be to formulate new research problems.

The three research areas lend themselves very well to participation by students and researchers at all levels. Undergraduate students can explore the concepts by analyzing the simplest models and by computer simulation. Two of the current graduate students are working on related problems for their dissertation research, and several postdocs will be working with the participating faculty on projects with one of the themes of the group (including Visiting Assistant Professor Ueltschi, and Visiting Postdoctoral Fellow Michael, who start their appointments in the fall of 2001).

A miniature version of this Research Focus Group has been in operation over the past two summers and academic year. The regular participants were eight undergraduate students majoring in physics and/or mathematics who worked on REU projects with Nachtergaele and Tracy during the summers of 2000 and 2001 (Bosse, Bunting, Clark, Guan, Pack, Puri, and Whitbeck), five graduate students (Abbott, Dieng, McCoskey, Starr, and Wolowski), two Visiting Research Assistant Professors (Spitzer and Bru), and one visiting postdoctoral fellow (Bolina). An informal seminar covered a variety of topics related to the current research projects of the participating faculty, postdocs, and the more advanced of graduate students, but the seminars were frequently attended by a broader group of faculty and students.

We expect that this VIGRE Research Focus Group will have approximately 20 participants.

3) *Research Focus Group in Mathematical Biology*.

Faculty participants: Craig Benham, Angela Cheer, Alexander Mogilner, Claudia Neuhauer.

This Research Focus Group will emphasize mathematical modeling of biological phenom-

ena on all levels of organization and complexity: from biomolecules to cells to populations. The main goal is to prepare affiliated students and postdocs to become active participants of the ongoing “quantitative” revolution in biology. The activities of the group will be centered around three research projects: structure and function of proteins and DNA (Benham and Mogilner), biofluid dynamics and physiology of cells and tissues (Cheer and Mogilner), and spatially explicit models of community dynamics in ecology (Neuhauser and Mogilner). These projects will allow participating students and postdocs to develop a broad, panoramic view of the modern mathematical biology.

Each of the four participating faculty mentors has 1 to 3 postdocs and a similar number of graduate students. In addition, about 10-15 students and postdocs from the Division of Biological Science who use mathematics in their research will participate in the group’s activities. Thus, the group will have 25-40 affiliated students and postdocs, plus 10-15 interested undergraduate students from the new Applied Mathematics major with emphasis on Computational Biology. Undergraduate students, with postdocs and advanced PhD students as mentors, will participate attend seminars and work on simple subprojects associated with ongoing research in the group. They will be able to explore the biomathematical concepts by analyzing the simplest models and performing computer experiments.

Examples of specific research topics: Mechanochemistry and models of molecular motors, Statistical mechanics of DNA, Fluid and elastic dynamics of endothelial cells and blood near the cell wall, and Hierarchy of models of perturbation of biological communities at different spatial scales.

The seminar speakers will be recruited mainly from UC Davis, UC Berkeley, Stanford, U. of Utah and other colleges in Northern California and neighboring Western states. Prominent scientists visiting Berkeley and Stanford will be invited to make a detour to Davis.

The reading group will pick ‘hot’ papers relevant to the group’s projects, from Nature, Science, PNAS, Biophysical J., J. of Theoretical Biology, etc. All students will read and think about a current paper for a week, and once a week the paper will be discussed.

Examples of suitable research projects for undergraduates include: Mesh generation for blood flow in artery, Kinetic equation models of molecular motor’s mechanochemical cycle, Monte Carlo simulations of DNA dynamics, Cellular Automata modeling of plant communities.

#### 4) *Discrete Mathematics.*

Faculty participants: Jesus De Loera, Greg Kuperberg, Anne Schilling, Roger Wets.

Combinatorial mathematics has quickly demonstrated its importance as a formal frame to study problems in computer science and information technology. Here at Davis, several faculty members actively pursue research and teaching in (1) Algebraic and Geometric Combinatorics (De Loera, Kuperberg, Schilling), (2) Stochastic Discrete Optimization and Integer Programming (De Loera, Wets). In the first area examples of research questions are: Enumeration of magic squares and contingency tables, the combinatorial properties of quantum algebras, triangulations of convex polytopes and oriented matroids, polynomial ideals related to chromatic polynomials in graphs, and the enumeration of alternating-sign matrices and mathematical physics. Under the second area there is an active study of

new combinatorial-algebraic techniques for solving integer programming problems, Gröbner bases and test sets in stochastic programming, Stochastic Network reliability, and algebraic generation of Lovasz-Schrijver cuts. Two research seminars (Optimization seminar and Combinatorics seminar) with very active participation from colleagues from CS, ECE, School of Management run weekly. Currently 6 PhD Students (Ahmed, De Groot, Lucero, Marsh, Singh, Yoshida), one postdoc (Hemmecke) as well as two minority undergraduate students (Garcia, Johnston) participate in the projects. Two sabbatical visitors (Onn and Santos), will join the activities.

Topics for seminar talks: The volume of the Birkhoff polytope, minimum weight planar triangulations, algebraic geometry techniques in stochastic optimization, parametric linear programming, graph theory and polynomial ideals, algorithms for computing Hilbert bases, lattice points in combinatorics and representation theory, polyhedral combinatorics and applications in optimization.

Possible choices for reading courses: Sebö, *Hilbert bases and a combinatorial Caratheodory type theorem*; Sturmfels, *Gröbner bases and convex polytopes*; Cox and Little O'shea, *Ideals, Varieties and Algorithms*; Stanley, *Combinatorics and commutative algebra*; Sagan, *The symmetric group: combinatorial algorithms and representation theory*; Rockafellar and Wets, *Variational Calculus*.