

Quiz 2 Solutions:

1) $\lim_{x \rightarrow c} f(x) = L$ means we can make $f(x)$

very close to L by taking x sufficiently close to c .

2) Find asymptotes, if any.

(i) $f(x) = \frac{x^2 + 1}{x^2 - 8}$.

Horizontal asymptote = at $y = 1$

Vertical asymptotes at ~~$x = \pm 2$~~ $x = 2$

(ii) $g(x) = \frac{x^2 - 1}{2x^2 - 8}$

Horizontal asymptote: $y = \frac{1}{2}$

Vertical asymptote: $x = \pm 2$

3) Does limit exist?

$$(i) \lim_{x \rightarrow 1} \begin{cases} 1-x & x \leq 1 \\ (x-1)^2 & x > 1 \end{cases}$$

Check left/right hand limits.

$$* \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} (1-x) = 0$$

$$* \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} (x-1)^2 = 0$$

Left and right limits agree so limit exists.

$$(ii) \lim_{x \rightarrow 0} \frac{|x|}{x}. \text{ Note: if } x > 0, \text{ then } \frac{|x|}{x} = \frac{x}{x} = 1$$

$$\text{and if } x < 0, \text{ then } \frac{|x|}{x} = \frac{-x}{x} = -1.$$

$$\text{So: } \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \lim_{x \rightarrow 0^-} -1 = -1$$

$$\lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} 1 = 1.$$

Left and right limits do not agree,

so limit does not exist.