Quiz Solutions: Quiz #3

Find intervals on which the following are continuous.

1) (i) $f(x) = \frac{2x-2}{x-1} = \frac{2(x-1)}{x-1}$

Not defined at $x=1$, so not continuous at $x=1$. Continuous on the intervals $(-\infty, 1) \cup (1, \infty)$

(ii) $g(x) = \begin{cases} x^2-4 & x<0 \\ 3x+1 & x \geq 0 \end{cases}$

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0} x^2-4 = -4$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0} 3x+1 = 1$

Right / left hand limits don't agree, so not continuous at $x=0$. Continuous on $(-\infty, 0) \cup (0, \infty)$.

(iii) $g(x) = \frac{1}{x^2+4}$

Since $x^2+4 > 0$ everywhere, $g(x)$ is continuous everywhere.

2) Find the derivative.

(i) $$\frac{d}{dx}\left[\tfrac{14}{3}\cdot x + \tfrac{5}{2}\right] = \frac{d}{dx}\left[\tfrac{14}{3}x\right] + \frac{d}{dx}\left[\tfrac{5}{2}\right]$$

$$= \tfrac{14}{3}\,\frac{d}{dx}[x] + 0$$

$$\boxed{= \tfrac{14}{3}}$$

(ii) Simplify: $\frac{t+1}{t^2} = \frac{t}{t^2} + \frac{1}{t^2} = \frac{1}{t} + \frac{1}{t^2} = \underline{\underline{t^{-1} + t^{-2}}}$

Differentiate: $\frac{d}{dt}[t^{-1} + t^{-2}] = \frac{d}{dt}[t^{-1}] + \frac{d}{dt}[t^{-2}]$

$$\boxed{= -1\cdot t^{-2} + -2\cdot t^{-3}}$$

(iii) $$\frac{d}{dr}\left[\tfrac{4}{3}\pi r^3\right] = \tfrac{4}{3}\pi\cdot\frac{d}{dr}[r^3]$$

$$= \tfrac{4}{3}\pi\cdot 3r^2 = \boxed{4\pi r^2}$$

(iv) $y = (2x-1)(2x+1) = 4x^2 - 1$

$$\frac{dy}{dx} = \frac{d}{dx}[4x^2 - 1] = \frac{d}{dx}[4x^2] + \frac{d}{dx}[-1]$$

$$= 4\cdot\frac{d}{dx}[x^2] + 0$$

$$= 4\cdot 2x\ \boxed{= 8x}$$

(iv) $t(x) = \frac{1}{\sqrt[3]{x^4}} = ((x^4)^{1/3})^{-1} = x^{-4/3}$

$\frac{d}{dx}[x^{-4/3}] = -4/3\, x^{-4/3-1}$

$\boxed{= -4/3 \cdot x^{-7/3}}$

3) Find equation for the tangent line to $y = 2x^2 + 10$ at $(0, 10)$.

Slope: $\frac{dy}{dx} = 4x$, so $\frac{dy}{dx}(0) = 4 \cdot 0 = 0$

Put into point-slope form - get

$(y-10) = 0 \cdot (x-0)$

or $\boxed{y = 10}$