Math 16-A: Quiz #5

Name:

1. Find $\frac{dy}{dx}$ in terms of x and y using *implicit differentiation.* (Partial credit for using another method).

i. $x^2y = \cos(x)$

$$\frac{d}{dx}[x^2y] = \frac{d}{dx}[\cos x]$$

$$2x \cdot y + x^2 \cdot \frac{dy}{dx} = -\sin x$$

$$\boxed{\frac{dy}{dx} = \frac{-\sin x - 2xy}{x^2}}$$

ii. $x^2 + 2y^2 = 4$

$$\frac{d}{dx}[x^2+2y^2] = \frac{d}{dx}[4]$$

$$2x + 2 \cdot 2y \cdot \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{4y}}$$

iii. $xy^2 - x^2y = 21$

$$\frac{d}{dx}[xy^2 - x^2y] = \frac{d}{dx}[21]$$

$$\left(x \cdot 2y \cdot \frac{dy}{dx} + 1 \cdot y^2\right) - \left(2xy + x^2\frac{dy}{dx}\right) = 0$$

$$2xy\frac{dy}{dx} - x^2\frac{dy}{dx} + y^2 - 2xy = 0$$

$$\boxed{\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}}$$

2. If I throw a rock in a pond, a large ripple will expand outward in concentric circles. Let $r(t)$, $C(t)$, and $A(t)$ denote the radius, circumference, and area of the circle (respectively) t seconds after the rock hits the water.

i. Suppose the radius is increasing at a constant rate of 2 meters/sec. How fast is the circumference increasing, in meters/second? (Remember: $C = 2\pi r$)

We are given: $\frac{dr}{dt} = 2$, always.

$C(t) = 2\pi \cdot r(t)$

$$\frac{dC}{dt} = \frac{d}{dt}(2\pi \cdot r(t)) = 2\pi \cdot \frac{dr}{dt} = \boxed{4\pi \ \frac{m}{s}}$$

ii. How fast is the area increasing when the radius is 3 meters? (Remember: $A = \pi r^2$).

$A(t) = \pi(r(t))^2$

$$\frac{dA}{dt} = \frac{d}{dt}[\pi r(t)^2] = \pi \cdot 2 \cdot r \cdot \frac{dr}{dt}$$

when $r=3$, get $\frac{dA}{dt} = 2\pi \cdot 3 \cdot 2 = 12\pi$

iii. Find $A''(3)$ (Hint: remember that $r'(t)$ is constant).

From (ii),

$$A'(t) = 2\pi r(t) \cdot \frac{dr}{dt} = 4\pi \cdot r(t)$$

Then $A''(t) = \frac{d}{dt}[4\pi r(t)]$

$$= 4\pi \cdot \frac{dr}{dt}$$

$$\boxed{= 8\pi}$$