Math 16-A: Quiz #4

Name: Solutions

1. The Chain Rule. For each function, **(i)** find $y = f(u)$ and $u = g(x)$ so that $y = f(g(x))$ and **(ii)** use the chain rule to find $\frac{dy}{dx}$ (partial credit if you do not use the chain rule).

(a) $y = (5x-2)^4$

(i) $y = u^4$, $\frac{dy}{du} = 4u^3$

$u = 5x-2$ $\frac{du}{dx} = 5$

(ii) $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 4u^3 \cdot (5)$

$= 4(5x-2)^3 \cdot 5$

$\boxed{= 20(5x-2)^3}$

(b) $y = \sqrt[3]{x^{-2}}$.

(i) $y = u^{\frac{1}{3}}$ $\frac{dy}{du} = \frac{1}{3}u^{-\frac{2}{3}}$

$u = x^{-2}$ $\frac{du}{dx} = -2x^{-3}$

(ii) $\frac{dy}{dx} = \left(\frac{1}{3}u^{-\frac{2}{3}}\right) \cdot (-2x^{-3})$

$\boxed{= \frac{1}{3}(x^{-2})^{-\frac{2}{3}} \cdot (-2x^{-3})}$

(c) $y = 4\cos(4x^4 + 3x + 9)$

(i) $y = 4\cos(u)$, $\frac{dy}{du} = -4\sin(u)$

$u = 4x^4 + 3x + 9$ $\frac{du}{dx} = 16x^3 + 3$

(ii) $\frac{dy}{dx} = (-4 \cdot \sin(u)) \cdot (16x^3 + 3)$

$\boxed{= -4\sin(4x^4 + 3x + 9)(16x^3 + 3)}$

(d) $y = \frac{1}{\sqrt[4]{x^2 - 3x + 4}}$

(i) $y = u^{-\frac{1}{4}}$ $\frac{dy}{du} = -\frac{1}{4}u^{-5/4}$

$u = x^2 - 3x + 4$ $\frac{du}{dx} = 2x - 3$

(ii) $\frac{dy}{dx} = \left(-\frac{1}{4}u^{-5/4}\right) \cdot (2x-3)$

$\boxed{= -\frac{1}{4}(x^2 - 3x + 4)^{-5/4} \cdot (2x-3)}$

2. The temperature of food in a refrigerator after t hours is given by

$$F(t) = 10\left(\frac{4t^2+100}{t^2+10}\right)$$

(a) Find $F'(t)$.

Use quotient rule:

$$F'(t) = 10\left[\frac{(t^2+10)\frac{d}{dt}(4t^2+100) - (4t^2+100)\frac{d}{dt}(t^2+10)}{(t^2+10)^2}\right]$$

$$= \underbrace{10\left[\frac{(t^2+10)(8t)-(4t^2+100)(2t)}{(t^2+10)^2}\right]}_{\text{okay to leave like this...}} = \underbrace{10\left[\frac{-120\cdot t}{(t^2+10)^2}\right]}_{\text{... but happens to simplify nicely. Useful for next problem.}}$$

(b) Find $F'(0)$ and $F'(1)$. What does $F'(t)$ represent physically?

$$F'(0) = 10\left[\frac{-120\cdot 0}{10^2}\right] = 0$$

$$F'(1) = 10\left[\frac{-120}{11^2}\right] = \frac{-1200}{121}.$$

$F'(t)$ is the rate at which the temperature of the food decreases.

$F'(0)=0$ means at first, the temp. of the food isn't changing.

$F'(1) = -\frac{1200}{121} \approx -10$, means after 1 hr, the temp. decreases at 10 degrees/hr.

Extra credit: Intuitively, we know that the temperature of the food will continue to drop until it's as cold as the refrigerator. If this F accurately models the temperature of the food, how cold is the refrigerator? How do you know?

As t increases, $F(t)$ approaches the temp. of the refrigerator, T_R.

So we take a limit as $t \to \infty$ to find T_R:

$$T_R = \lim_{t\to\infty} F(t) = \lim_{t\to\infty} 10\left[\frac{4t^2+100}{t^2+10}\right] = 40.$$

So the refrigerator is 40 degrees.