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Statement of Research Interests and References

My research interests include fluid dynamics, partial differential equations arising from physical problems, and large dynamical systems in general. Using techniques such as multiple-scales, weakly nonlinear asymptotics, small scale numerical computation and large direct numerical simulations, my work is aimed at isolating the essential processes underlying natural phenomena. My goal is to choose applied problems on which the mathematics can have the greatest impact. In particular, it is important to use careful analysis of physical models in order to fill in the gaps which arise in large numerical simulations or to scale up the results of smaller numerical models.

The lack of understanding of the effect of the tropics on the Earth's climate is a major impediment to increasing predictability in the midlatitudes, the latitude band which extends from the tropics to about 60° . In a series of three papers with A. Majda of the Courant Institute of Mathematical Sciences [3, 4, 6], we developed a theory for the nonlinear interaction on intraseasonal ($\sim 10 - 20$ day) time scales of two different types of waves, which are driven by the gradient of the Coriolis force at the equator. The resultant coupled nonlinear dispersive equations for the amplitudes of interacting wave packets are novel both from the perspective of the atmospheric sciences and from a more general mathematical setting. They have a Hamiltonian structure and admit analytic solitary wave solutions. I will briefly outline the context and derivation of this theory and discuss additional atmospheric and mathematical issues which remain to be pursued.

Also in collaboration with A. Majda, I have developed an asymptotic theory of planetary scale phenomena in the tropical atmosphere [1, 2]. This is the first multiscale understanding of the Madden-Julian oscillation, a phenomenon described as the "holy grail" of tropical meteorology. Though the Madden-Julian oscillation has been observed in nature, it has yet to be convincingly simulated in large scale computations, the culprit being the lack of resolution of the correct scales of the interaction. Using a multiple-scale perturbation theoretic framework, we are able to show which aspects of the observed smaller scale phenomena in the tropical atmosphere give rise to planetary scale organized flows seen in the Madden-Julian Oscillation.

In addition to my work with A. Majda, there are three major projects which I continue to pursue. In an ongoing collaboration with P. Kramer and Y. Lvov at Rensselaer Polytechnic Institute, we are studying weak turbulence predictions of the statistical equilibrium in the Fermi-Pasta-Ulam problem [7, 8]. Secondly, my work on double-diffusive convection has involved laboratory experiment [12], asymptotics [11], and a full scale direct numerical simulation as part of my doctoral dissertation [9]. Thirdly, working with N. Lebovitz at the University of Chicago and P. Morrison at the University of Texas at Austin, we have developed a theory of the Riemann ellipsoids using an exact reduction to a system of ODE's from the Hamiltonian description of incompressible fluids. Finally, I have begun work on the interaction among multiple scales in the tropics in the presence of moisture and moist convection. Moisture has a significant impact on the atmosphere and the description of moist convection results in novel PDE's to describe the atmosphere, for example, free boundary problems.

The Nonlinear Interaction of Equatorial Baroclinic and Barotropic Rossby Waves

In a series of three papers with A. Majda, we studied the connection of the tropics to the midlatitudes

through the interaction of two sets of waves which are peculiar in that they are driven by the gradient of the Coriolis force near the equator. These are the baroclinic Rossby waves, which are confined near the equator and respond to heating, and equatorial barotropic Rossby waves which extend onto large weather systems in the midlatitudes. The ideal, incompressible fluid equations in the presence of the Coriolis force, which changes sign at the equator, are the primitive equations of tropics. Both sets of Rossby waves are solutions to the linearized primitive equations that are vortical in nature, propagate westward, and are nearly dispersionless at long longitudinal wavelengths.

Starting with the primitive equations, we have constructed a long wave theory for anisotropically scaled longitudinal waves in the equatorial troposphere. This theory retains the lowest-order nonlinear interactions and dispersion. Since it describes intermediate time evolution of interacting waves with large longitudinal extent, this theory is itself interesting from the perspective of mid-range (~ 2 week) weather forecasting, and merits further numerical study. On planetary scales tropospheric flows have wind velocities of 5-10 meters/second which are much less than the wave propagation speed of 50 meters/second. This makes them weakly nonlinear. The presence of advection and vortex stretching in the long wave theory allows the possibility of a resonant interaction between the two Rossby wave packets, which must be removed by applying the Fredholm alternative in a matched asymptotic expansion which balances dispersion and nonlinearity. The asymptotics of the dispersion balanced nonlinearity is the same matched asymptotic limit which is used to derive the Korteweg-de Vries (KdV) equations from the shallow water equations and results in the system of nonlinear PDE's

$$\begin{aligned} A_t - D A_{xxx} + (AB)_x &= 0 \\ B_t - B_{xxx} + A A_x &= 0 \end{aligned}$$

which govern the amplitudes of a the two wave packets with relative dispersion, $D \approx 1$.

We have shown that these equations have at least one Hamiltonian structure with a bracket akin to one of those of the KdV equation, a conserved energy and Hamiltonian, and that they admit some analytic solitary wave solutions. An interesting open question remains as to whether these equations contain more solitary waves solutions and the possibility that they might form an integrable system.

We have elaborated this theory by allowing the possibility of more general background flow and damping through boundary layer drag against the bottom of the troposphere. Boundary layer dissipation is of particular importance for the atmosphere and results in an interesting, degenerate linear operator coupling the above two types of Rossby waves, thereby enhancing tropical/midlatitude energy exchange. Since the actual atmospheric boundary layer is also a source of moisture for the free troposphere, the addition of a moist boundary layer is an important future extension of this work.

Multiscale Asymptotics for the Equatorial Troposphere and the Madden-Julian Oscillation

The Madden-Julian oscillation is a wave of organized convective activity which begins over Indonesia, propagates eastward with a velocity of about 5 ms^{-1} , ends over the tropical eastern Pacific, and has a period of about 40-50 days. It is latitudinally confined to the tropics, but longitudinally global in nature, with lagged correlations in wind velocity extending from the east coast of Africa to the west coast of South America. Among other features, it is characterized by a “westerly wind burst” at the base of the troposphere near the equator which is a dramatic reversal of the easterly trade winds that tend to occur there. Since it occurs on intraseasonal time scales, the Madden-Julian Oscillation is a major component of intraseasonal weather variability in the tropics, and our lack of theoretical understanding of this phenomenon is a major limitation in medium range midlatitude weather predictability. Specific incidences of the Madden-Julian Oscillation have also been observed to shut off an El Niño (through the interaction with the ocean surface) and affect the average angular momentum of the Earth (changing the length of day by about 0.5 ms). The nature of these interactions is still poorly understood.

The linear dynamics of the primitive equations show that tropical waves are latitudinally confined to distances of a few times the equatorial synoptic scale ($\sim 1500 \text{ km}$) about the equator. Majda & Klien developed

an asymptotic theory for the tropical primitive equations which describes the coupling of planetary to smaller scales on time scales of several days and exploits the small ratio of the latitudinal extent of the equatorial wave guide to the longitudinal planetary scale, $\epsilon \sim 1500 \text{ km} : 40000 \text{ km}$. Additionally, wind speeds averaged over the large scale result in nonlinear advection terms which are order ϵ times the linear, wave propagation terms. In our model of the Madden-Julian Oscillation [1, 2], we take the important features of the observational record on the synoptic scales and, using this multiscale asymptotic framework, calculate the resultant planetary scale flows: ours is the first model of Madden-Julian Oscillation whose results are in excellent agreement with the observational record.

The main driving force in the tropics is the latent heat released when moisture condenses to clouds and eventually falls to the ground as precipitation. An essential piece of our understanding of the Madden-Julian Oscillation has involved creating an analytic description of this heating. We assume that the tropical heating varies on the two longitudinal length scales, the planetary and synoptic scales, and has a planetary scale mean which is $O(\epsilon)$ times the small scale fluctuations. The result is a flow which also consists of a planetary scale mean and fluctuations. The fluctuations of the flow on the smaller scale are determined by the balanced dynamics of the asymptotic theory forced by the heating fluctuations. The large scale means of the flow are determined from the asymptotic theory by linear equatorial wave dynamics, which have the additional constraint that latitudinal pressure gradients balance the Coriolis force. In the theory, not only are planetary scale flows forced by the planetary scale mean heating, but also by the average upscale flux of momentum and temperature from the small scales.

Our model has the advantage that the analytic specification of the heating profile yields an analytic expression for the small scale flows and their resultant upscale fluxes of momentum and temperature. We clearly show that a particular structure of the heating on the small scales (upper troposphere in the east and lower troposphere in the west) yields the upscale fluxes which are necessary to drive the observed planetary scale structures. The planetary scale flows, themselves, are governed by quasi-linear time dependent PDE's and are solved by an algorithm I developed using an appropriate basis for long waves in the equatorial troposphere. In this theory, we provide the first clear multiscale understanding of the Madden-Julian Oscillation which, when combined with the separation of the planetary scale forcing into mean and upscale fluxes, and the analytic description of these fluxes, allows us to clearly determine which features on the small scales are required in order to attain the observed planetary scale structures.

The remaining question is, how do planetary scale flows organize and modify the smaller scale latent heating profile which sustains them. This is the important ongoing piece of our research on the Madden-Julian Oscillation and I expect it to be a source of several interesting projects for the future. In particular this feedback requires the addition of the complicated and mathematically rich processes of moisture and moist convection, allowing for interactions across even more spatial and temporal scales.

Weak Turbulence and Statistical Equilibrium in the Fermi-Pasta-Ulam Problem

One of the very first uses of electronic computing machines was Fermi, Pasta, and Ulam's simulation of wave propagation in a weakly nonlinear lattice model (FPU). It is well known that the redistribution of energy among the normal modes of the system which FPU expected to observe did not occur in their calculation. Instead, the system displayed the regular, periodic behavior characteristic of integrable systems. Though this discovery shifted attention for several decades to the near-integrable nature of FPU, in the last two decades, the FPU model with more degrees of freedom has once again been utilized as a test model for numerically illustrating and exploring standard concepts in statistical mechanics.

There exist parameter regimes of the FPU system for which it is weakly nonlinear but chaotic, and it is in these regimes that one can hope to connect the outcome of direct numerical simulations with statistical mechanical concepts, such as relaxation to equipartition, entropy production, universal behavior of statistical functions and virial relations. The FPU model is one of the simplest and most natural nonlinear models for statistical mechanics which can be conceived, and is therefore a natural testbed.

In my work with Lvov and Kramer at Rensselaer Polytechnic Institute [8, 7], we use the FPU model to scrutinize weak turbulence theory, a nonequilibrium statistical mechanical theory which attempts to describe the dynamical energy transfer among normal modes in a weakly nonlinear, dispersive, extended system. Combining numerical simulations and asymptotics, we use the FPU model to examine the assumptions underlying weak turbulence theory, namely quasi-Gaussianity of the moments of the normal modes and random phases of the waves. Our scaled weak turbulence theory predicts that there is a long lived “quasi-equilibrium” in the FPU equations. In a model of N oscillators with nonlinearity measured by ϵ , our theory predicts that the spectral range of the modes in equilibrium scales as $N\epsilon^{1/2}$ and occurs over time scales of $\epsilon^{-3/2}$. For large enough systems, our numerics bear out these predictions, but the breakdown of this long-lived transient to a state of equipartition among all of the modes is not yet well understood.

In the ongoing phases of this project we are carefully forcing and damping the FPU model in order to study the “flux solution” regime of weak turbulence theory. These solutions correspond to fluxes of energy in phase space and are the weakly nonlinear analog of the Kolmogorov spectrum of strong turbulence. Though these solutions are often invoked as models of physical problems, their physical relevance and regimes of applicability are in dispute. By studying the time evolution of the energy cascade we are trying to determine whether the dynamical system actually visits any of the flux solutions and if so, which does it select.

Double-Diffusive Convection and Astrophysical Semiconvection

As part of my graduate dissertation work and in other papers [9, 12, 11], I investigated the thermal convection in a fluid layer in the presence of a stabilizing density gradient such as salt in the ocean or helium in the hydrogen convective region of a star. This is often referred to as the diffusive regime of double diffusive convection and is a major impediment to vertical mixing in several physical situations.

In [11] we developed the weakly non-linear theory above the onset of the oscillatory instability which arises in double-diffusive convection. The equations derived therein are similar to coupled Ginzberg-Landau equations and describe the evolution of the stream function, stabilizing and destabilizing components of the density and have interesting solutions which exhibit convective bursts.

From the physical perspective, the strongly nonlinear regime is more interesting, and I explored this regime both with an experiment [12] and through a direct numerical simulation [11] using a finite difference, pseudo-spectral parallel code. Double-diffusive convection is known to develop horizontal strata with density jumps across their interfaces and strong convection between the interfaces. The strata are long-lived transients which greatly hinder the vertical mixing of the fluid. My work investigated and attempted to characterize the growth of internal interfaces in the presence of convection and determine a vertical mixing rate for the stabilizing component of the density. With F. Rubini (joint Universities of Florence and Chicago) we are developing a more tailored numerical scheme in order that we can investigate the mixing properties of interfacial waves in the presence of convection.

The Hamiltonian Formulation of the Riemann Ellipsoids

I have developed a description of the Riemann ellipsoids with N. Lebovitz (Chicago) and P. Morrison (Texas, Austin) using the moment reduction formalism for continuous Hamiltonian systems with the added stipulation of restricting our dynamics to the space of incompressible fluid flows. In our description, we derive a Hamiltonian version of the equations governing the classical self-gravitating exact solutions of the incompressible Euler equations, known as the Riemann ellipsoids. The resultant dynamical system is 15 dimensional with three Casimirs (conserved quantities which are due to the coordinates used to describe the phase space) which arise naturally from the reduction: the total circulation, volume of the figures and divergence of the velocity field. Furthermore, we relate this derivation to the Dirac bracket projection of Hamiltonian mechanics wherein the conserved quantities are specified at the outset.

Our description of the Riemann ellipsoids is novel in that it has the energy as its Hamiltonian while the constraints of incompressibility are built into the dynamics. Furthermore, in our theory it is straightforward

to add external and internal forces such as magnetic fields, which would increase the dimension of the dynamical system but retain its phase space structure. The Hamiltonian description of the Riemann ellipsoids is unique in that, as a dynamical system, it is neither small nor large and therefore both a statistical dynamics description and the classical stability theory are interesting.

Multiple scales for moist tropospheric dynamics

Multiscale asymptotics for the equations of a moist atmosphere have barely been explored, and new numerical methods will be required in order to study them; for example, in one limit of moist atmospheric dynamics, the propagation of precipitation fronts is described by a free boundary problem. Furthermore the hierarchy of scales is immense, from planetary, to organized storm systems, to storm clusters, to individual storms, and each of these phenomena interact with and affect the top layer of the ocean. With the wealth of observations and the current power and limitations of large scale numerical computation, I foresee that careful analysis, modeling and smaller scale computations of the equations of the equatorial troposphere remains an interesting and highly applicable field of research.

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