

## Practice Midterm 2

### Problem 1 (10 points)

True or False? Circle one.

T/F For any functions  $F$  and  $G$ ,  $\text{Dom}(G \circ F) \subseteq \text{Dom}(F)$ .

T/F For any functions  $F$  and  $G$ ,  $\text{Dom}(G \circ F) \subseteq \text{Dom}(G)$ .

T/F Every relation that is symmetric and transitive is also reflexive.

T/F The relation  $\{(1, 2), (2, 3), (3, 1)\}$  is transitive.

### Problem 2 (10 points)

There is a natural correspondence between equivalence relations and partitions. If  $A$  is a set and  $R$  is an equivalence relation on  $A$ , then  $A/R$ , the set of  $R$ 's equivalence classes, is a partition of  $A$ . Prove or disprove: If  $R$  and  $S$  are two equivalence relations on  $A$ , then  $A/R \cap A/S$  is a partition of  $A$ .

### Problem 3 (10 points)

Prove that the relation " $\equiv_7$ " is an equivalence relation.

### Problem 4 (10 points)

Consider the relation  $R$  on  $\mathbb{N}$ , defined by:  $xRy$  iff  $x$  and  $y$  have a common factor greater than 1. Is this relation reflexive? Symmetric? Transitive?

### Problem 5 (10 points)

Prove that if  $f : A \xrightarrow{1-1} B$  and  $g : B \xrightarrow{1-1} C$ , then  $g \circ f$  is 1-1. Does this imply that  $g \circ f : A \xrightarrow{1-1} C$ ?

### Problem 6 (10 points)

Let  $f(x) = \frac{2x+1}{x-1}$ . Find the domain of  $f$ , assuming that it is the largest possible subset of  $\mathbf{R}$ . Find the range of  $f$ . Find an expression for  $f^{-1}(x)$ . What are the domain and range of  $f^{-1}$ ?

### Problem 7 (10 points)

Find an example of a function  $f : A \rightarrow B$  that is 1-1, and a function  $g : B \rightarrow C$  that is NOT 1-1, such that  $g \circ f$  is 1-1, or prove that this is impossible.