

Midterm 1 Review Packet for Math 12

This is intended as a study guide. The problems on the real test will not be the same as these, although they might be similar. In addition to working through these problems, you should also review as many homework problems as possible.

Remember that you will not have calculators on the midterm, so try to do these problems without a calculator.

Problem 1 If possible, find:

- an integer that is not a natural number
- a real number that is not rational
- a natural number that is not rational
- a rational number that is not an integer
- an integer that is also rational

Problem 2 What is larger, $2 + \sqrt{3}$ or $3 + \sqrt{2}$?

Problem 3 Simplify $|x - 4| + |x + 1|$, given that $-1 < x < 4$.

Problem 4 (a) The inequality $x^2 + 3 \leq 2$ has no solution. Why not? Carefully explain.

(b) Write an inequality of your own that has no solution.

(c) Can you write an inequality that has exactly one solution? Exactly two?

Problem 5 “The water level of Rollins Lake is always about 2130 feet above sea level, but is sometimes above or below this elevation by up to 3 feet.” Rewrite this sentence as an inequality, using absolute value, with the variable x representing the height (in feet) of the surface of Rollin’s Lake above sea level. Solve your inequality for x .

Problem 6 Write an inequality that expresses the fact that x is at least 5 units away from 12. Solve this inequality for x and write the solution set in interval notation.

Problem 7 Draw on the number line the solution set of the inequality $|x - 2| \geq 3$.

Problem 8 For each of the following expressions, find the domain and represent it in interval notation:

- $\sqrt{4 - x}$
- $\sqrt{4 - |x|}$
- $\sqrt{4 - x^2}$
- $\frac{(x+1)(x-3)}{(x-2)(x+4)}$
- $x^2 + 3x + 2$
- $\frac{1}{x^2+3x+2}$

Problem 9 Factor the following expressions as far as you can:

- $x^2 - a^2$
- $x^2 + a^2$
- $x^3 + 1$
- $x^4 - 16$
- $2x^3 - 9x^2 - 5x$

Problem 10 (a) Find all real solutions to the equation $\sqrt{x + 2} = x - 4$.

(b) Graph the functions $y = x - 4$ and $y = \sqrt{x + 2}$ on the same axes. What does your graph have to do with your answer to part (a)?

Problem 11 Calculate $(\sqrt{5} - 1)(\sqrt{5} + 1)$.

Problem 12 (a) Find the midpoint of (2,5) and (26,15).

(b) How far is the midpoint from each of the endpoints?

(c) Write the equation of the circle whose diameter has endpoints (2,5) and (26,15).

Problem 13 (a) How far is the point $(1, \sqrt{3})$ from the origin?

(b) Given that the point $(1, \sqrt{3})$ lies on a circle centered at the origin, what is the radius of that circle?

(c) What is the slope of the line segment from the origin to the point $(1, \sqrt{3})$?

(d) What equation describes the line tangent to the given circle at the point $(1, \sqrt{3})$?

Problem 14 You wrote the equation of a certain circle on your hand right before playing racquetball, and now one of the numbers is illegible. What remains of the equation is: $x^2 - 6x + y^2 - 8y + \clubsuit = 0$. Can you still find the center of the circle? (Here \clubsuit is the illegible smudge.)

Problem 15 Find the value of k for which the graph of $y = 2(x - 1)^2 + k$ passes through the origin.

Problem 16 Find the x - and y -intercepts of the line $5x - 2y = 10$, and plot the line.

Problem 17 Let $f(x) = \frac{|x|}{x}$ for all $x \neq 0$. Evaluate $f(x)$ for $x = \pm 1, \pm 2, \pm 3$. Graph $f(x)$.

Problem 18 (a) Write an equation for the horizontal line with y -intercept 3. Is this equation in slope-intercept form?

(b) Write an equation for the vertical line with x -intercept 1. Can you put this equation in slope-intercept form?

Problem 19 Write an equation for the line with y -intercept 3 and x -intercept 4. Put this equation in the form $ax + by = c$. Now divide by c to get an equation in the form $Ax + By = 1$. Do you notice anything interesting?

Problem 20 Consider the function $y = x^2$. Two points on it are $A(a, a^2)$ and $B(b, b^2)$. Find the slope of segment AB .

Problem 21 What is the average rate of change of $y = x^3 - 1$ on the interval $[-1, 2]$?

Problem 22 Without graphing it, determine whether the graph of $x^2 + y^4 - 3y^2 = 1$ is symmetric with respect to the x -axis, the y -axis, and/or the origin.

Problem 23 (a) For what value of x is the value of $f(x) = (x + 1)^2 + 3$ as small as possible?

(b) What is the minimal value of f ?

(c) What is the range of f ?

Problem 24 Graph $y = 2 - |x + 1|$. What are the coordinates of its vertex?

Problem 25 Find the domain, range, horizontal asymptote, and vertical asymptote of $y = \frac{2x+4}{x-1}$.