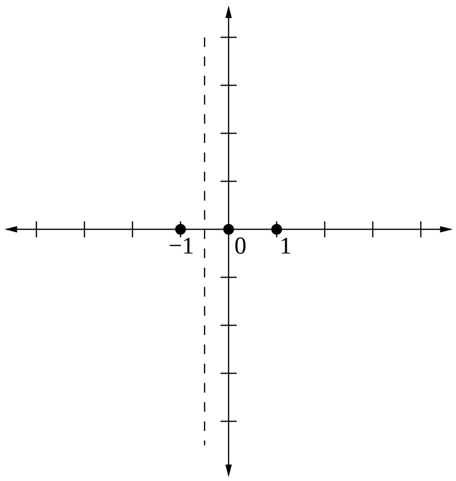
This is another example of how to graph a rational function.

Consider the rational function

$$y = \frac{x(x-1)(x+1)}{(x+\frac{1}{2})^2}.$$

The roots of the numerator are $x = -1, 0, 1$. These are the choices of x that make the whole function equal zero; they are also the x-intercepts of the graph of this function. The (double) root of the denominator is $x = -\frac{1}{2}$. For this value of x, the function is undefined; this corresponds to a vertical asymptote. So far, we have this much of the picture:



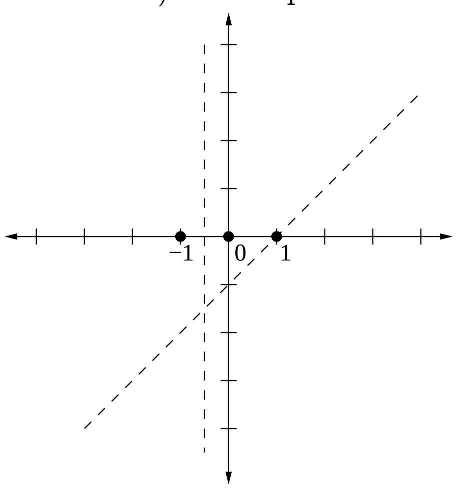
Now we compare the degrees of the numerator and denominator. The numerator is $x(x-1)(x+1) = x(x^2-1) = x^3 - x$, a cubic. The denominator is $(x+\frac{1}{2})^2 = x^2 + x + \frac{1}{4}$, a quadratic. So, in a sort of I-can't-find-my-glasses sense, this rational function will behave like a polynomial of degree $3-2=1$. That is, a line with some slope. That is, a slant asymptote. To find the slant asymptote, use long division. (Long division can also reveal horizontal asymptotes, but we have faster shortcuts for that.) The thing is, I don't know how to typeset long division. You should do the long division at home that confirms what I'm about to say, namely:

$$\begin{aligned}
x^3 - x &= (x-1)(x^2+x+\frac{1}{4}) - \frac{x-1}{4} \\
\frac{x^3-x}{x^2+x+\frac{1}{4}} &= x - 1 - \frac{x-1}{4(x^2+x+\frac{1}{4})} \\
&= x - 1 - \frac{x-1}{4x^2+4x+1} \\
&\quad \text{Divide top and bottom (of the last term) by } x\text{:} \\
&= x - 1 - \frac{1-\frac{1}{x}}{4x+4+\frac{1}{x}} \\
&\approx x - 1 - \frac{1}{4x+4} \quad \text{(for large } x\text{)} \\
&\approx x - 1 \quad \text{(for large } x\text{)}
\end{aligned}$$

Those two approximations use the fact that when we divide 1 by a large number, we get a small answer. So for large x, we can ignore the $1/x$ terms in the big fraction, and then we can ignore the fraction altogether. This gives us our slant asymptote, $y = x - 1$. We should check for places where our graph might cross this asymptote. So, we set the two functions equal to each other and solve for x:

$$
\begin{aligned}
\frac{x^3 - x}{x^2 + x + \frac{1}{4}} &= x - 1 \\
x^3 - x &= (x-1)(x^2 + x + \frac{1}{4}) \\
\underline{x^3} - x &= \underline{x^3} + x^2 + \frac{x}{4} - x^2 - x - \frac{1}{4} \\
\underline{-x} &= x^2 + \frac{x}{4} - x^2 \underline{-x} - \frac{1}{4} \\
0 &= \underline{x^2} + \frac{x}{4} \underline{-x^2} - \frac{1}{4} \\
0 &= \frac{x}{4} - \frac{1}{4} \\
\frac{x}{4} &= \frac{1}{4} \\
x &= 1
\end{aligned}
$$

Thus, the only point where our graph crosses the slant asymptote is at the point $x = 1$, which (by coincidence) shows up when we add the asymptote to our graph:



Now it's time to see how our function acts near certain "sensitive points". To show you as many examples of this as possible, I'll use the factored form of our function:

$$y = \frac{x(x+1)(x-1)}{(x + \frac{1}{2})^2}.$$

We see that interesting things will happen at $x = 0, -1, 1, -\frac{1}{2}$.

Near $x = 0$, we approximate y by substituting 0 for x in all quantities except the x factor upstairs:

$$y \approx \frac{x(0+1)(0-1)}{(0+\frac{1}{2})^2} = \frac{-x}{1/4} = -4x.$$

That is, near $x = 0$, our graph should cross the x-axis like a line of slope -4.

Near $x = -1$, we approximate y by substituting -1 for x in all quantities except the $x + 1$ factor upstairs:

$$y \approx \frac{-1(x+1)(-1-1)}{(-1+\frac{1}{2})^2} = \frac{2(x+1)}{1/4} = 8(x+1).$$

That is, near $x = -1$, our graph should cross the x-axis like a line of slope 8.

Near $x = 1$, we approximate y by substituting 1 for x in all quantities except the $x - 1$ factor upstairs:

$$y \approx \frac{1(1+1)(x-1)}{(1+\frac{1}{2})^2} = \frac{2(x-1)}{9/4} = \frac{8}{9}(x-1).$$

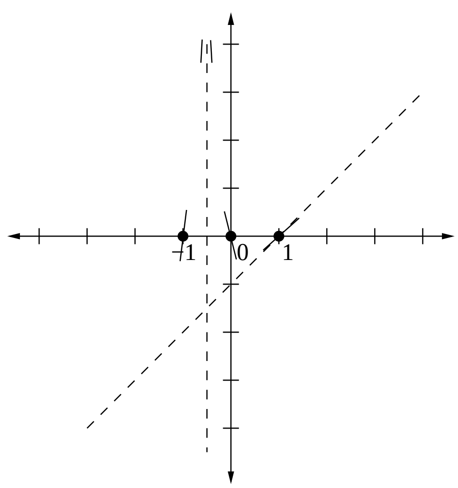
That is, near $x = 1$, our graph should cross the x-axis like a line of slope 8/9.

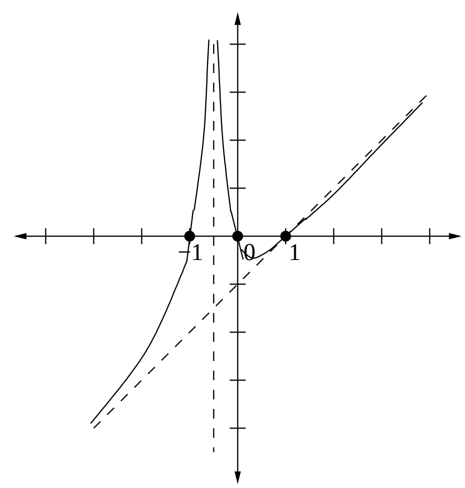
Finally, near $x = -1/2$, we approximate y by substituting -1/2 for x in all quantities except the $x + \frac{1}{2}$ factor downstairs:

$$y \approx \frac{-\frac{1}{2}(-\frac{1}{2}+1)(-\frac{1}{2}-1)}{(x+\frac{1}{2})^2} = \frac{(-\frac{1}{2})(\frac{1}{2})(-\frac{3}{2})}{(x+\frac{1}{2})^2} = \frac{3/8}{(x+\frac{1}{2})^2}.$$

That is, near $x = -1/2$, our graph should have two branches flying off the top of the graphing window, kind of like the graph of $y = \frac{1}{x^2}$. (On your own, try graphing $y = \frac{1}{x^2}$, $y = \frac{1}{(x+\frac{1}{2})^2}$, and $y = \frac{3/8}{(x+\frac{1}{2})^2}$.)

Below left, see our graph as it now looks. If we fill in the missing parts, we get the graph on the right.





This is the graph Mathematica made: