Homework 1

1.8.8. The most basic way to distinguish seatings is to treat two seatings in which any person occupies a different seat as distinct. In this case, there are 12 people that occupy 12 seats, so there are 12 places for the first person, 11 places for the second person, 10 places for the third person, and so on. This yields 12!, the number of permutations of the 12 people.

We might instead say that two seatings are the same if they only differ by rotating one of the tables or interchanging the tables. This interpretation would be useful if we were primarily concerned with who was sitting next to whom. In this case, we can define a "head seat" for each table and a "direction" that is clockwise or counterclockwise in which to seat people from the head. There are $6 \cdot 6 = 36$ configurations obtained by rotating the heads of the tables and $2 \cdot 2 = 4$ choices for the directions of the tables. Each of these yields two configurations obtained by switching the tables, for a total of $2 \cdot 36 \cdot 4 = 288$ configurations. Therefore, each seating is counted 288 times in the set of all permutations of the 12 people. This yields

$$\frac{12!}{288} = 1,663,200$$

seatings.

1.8.24. Since $10 = 2^{\log_2 10}$ where $\log_2 10 = 3.321928\cdots$, we have

$$10^{100} = \left(2^{\log_2 10}\right)^{100} = 2^{100 \log_2 10} = 2^{332.1928\cdots}$$

so 10^{100} has 333 digits when written in base 2.

1.8.25. There are 50 ways to choose the first capital to visit, 49 ways to choose the next capital, etc. Hence, there are a total of

$$50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 = 254,251,200$$

possible tours.

1.8.29. If we didn't have to worry about using each color at least once, the problem would be straightforward. There are 3 choices of color for the first object, 3 choices for the second object, and so on. This yields 3^n possible colorings. Unfortunately, some of these colorings will not use all three colors. However, these invalid colorings can be counted and then subtracted.

Suppose that the three colors are red, green and blue. Then, a coloring using two colors is invalid if it uses only red and green, or only red and blue, or only green and blue. There are $3 \cdot 2^n$ such colorings. The only other possibility is that a coloring is invalid because every object is assigned the same color. There are 3 such colorings, and these are counted twice in the formula above, so we must subtract them.

Hence, there are a total of

$$3^n - (3 \cdot 2^n - 3)$$

ways to color n objects with 3 colors such that every color is used at least once.

2. Among the integer numbers $1, 2, \ldots, 10^{10}$, are there more of those containing 9 in their decimal notation or those with no 9?

There are exactly 9^{10} strings of length 10 that are made up of the 9 digits $0, 1, \cdots, 8$. We can view each of these strings as an integer between 0 and 9,999,999,999 by ignoring any leading 0 digits. Hence, there are exactly $9^{10} = 3,486,784,401$ integers between 1 and 10,000,000,000 that do not contain a 9. (Notice that 0 does not count, but 10,000,000,000 does.) Since there are 10^{10} integers in all, the number of integers with at least one 9 in their decimal notation is $10^{10} - 9^{10} = 6,513,215,599$. Hence, there are more numbers containing a 9.

This may be surprising, because the result does not necessarily hold for smaller lengths; for example, there are *more* numbers between 1 and 100 that do *not* contain 9 than those which do.