Homework 3

0. Problems from sections 3 and 4:

3.8.10, 4.3.5, 4.3.7, 4.3.13.

1. Find a generating function to determine the number of different ways to make n cents from pennies, nickels, dimes and quarters. How would you determine the number of ways to make a dollar out of these coins?

2. Prove the identity

$$\sum_{k=0}^{n} \binom{n}{k}\binom{k}{m} = \binom{n}{m} 2^{n-m}.$$

3. How many distinct terms are there in the expansion of

$$(x_1 + x_2 + \cdots + x_m)^n \quad ?$$

For example, when $m = 2$ there are $n + 1$ terms corresponding to $k = 0, 1, \ldots, n$ in the expansion $\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$. How many total terms are there in the expansion? For example, when $m = 2$ we have seen that there are 2^n total terms.

4. Use the generating function method from lecture to find a closed formula in terms of n for the sequence given by

$$a_n = 3a_{n-1} + 1, \qquad a_0 = 0.$$

Some initial terms of the sequence are $0, 1, 4, 13, 40, 121, 364, \ldots$.