

Homework 4

0. Problems from sections 7 and 8:

7.3.9, 7.3.10, 8.5.3.

1. For each sequence A below, either find a graph whose vertex degrees are given by A , or prove that no such graph exists.

(a) $A = (3, 2, 2, 1, 1)$.

(b) $A = (4, 3, 2, 1, 0)$.

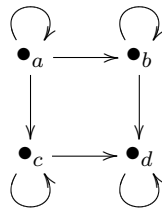
(c) $A = (5, 5, 4, 4, 4)$.

(d) $A = (3, 2, 2, 1)$.

2. Prove that in any graph, there are two vertices with the same degree.

3. Suppose a graph has $m \geq 2$ edges. What is the smallest number of vertices that the graph can have?

4. Consider the following graph G :



Let f_n denote the number of directed walks of length n in G . Use the adjacency matrix method from lecture to find a recurrence for f_n .

For example, the number of walks of length 1 is the number of edges, so $f_1 = 8$. The number of walks of length 2 turns out to be $f_2 = 14$:

$aaa, aab, aac, abb, abd, acc, acd, bbb, bbd, bdd, ccc, ccd, cdd, ddd$.

(Hint: Write down the adjacency matrix A for G and then find the determinant of $I - xA$.)